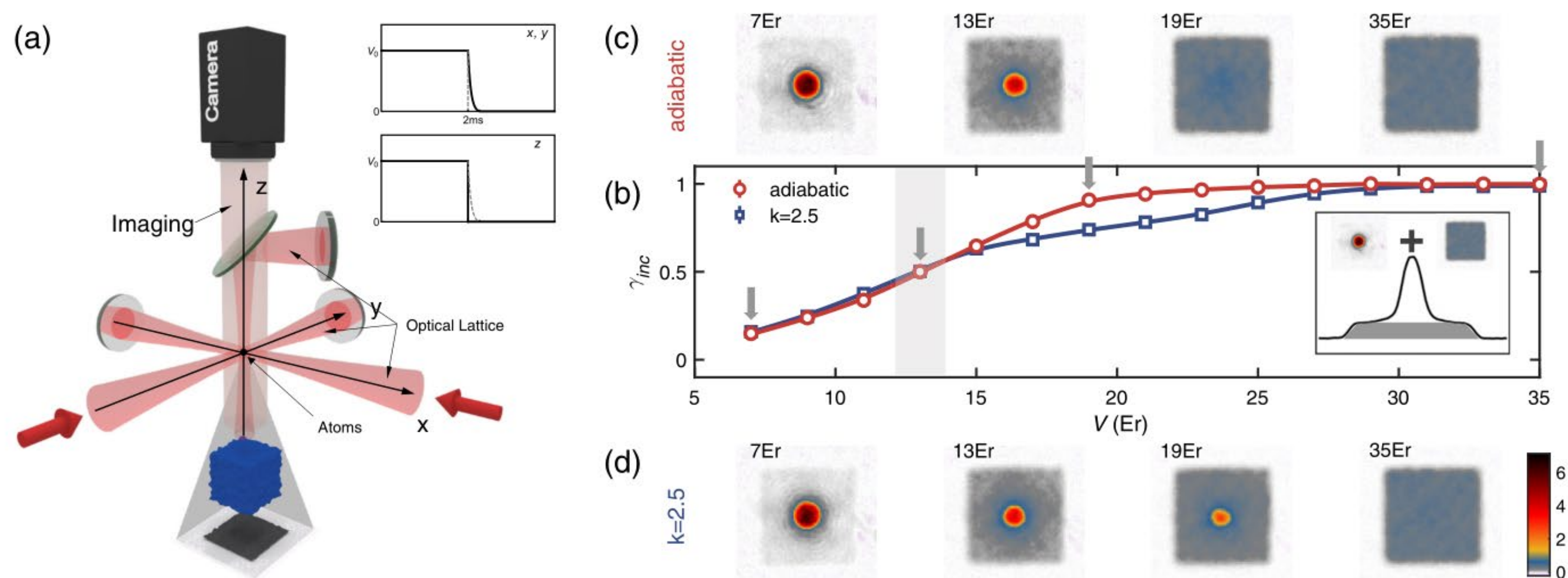


## Introduction

The quantum critical behaviors of many-body phase transition is one of the most fascinating challenging questions in quantum physics. We improved the band-mapping method to investigate the quantum phase transition from superfluid to Mott insulators, and we measured critical exponents of quantum phase transitions with Kibble-Zurek mechanism. Based on various observables, two different values for the same quantum critical parameter are observed. Beside the measurement of critical exponents of quantum phase transitions, we demonstrate the critical dynamics under dimensional crossover involving many-body phase transitions by continuously suppressing correlations and tunnelings along one direction of bulk materials. This provides a smooth connection from higher dimensions to lower dimensions based on intrinsic correlations instead of geometry tailoring. By measuring the non-adiabatic excitations, both critical scaling laws in 3D and 2D are observed and consistent with predictions. Besides, we find new scaling behaviors for intermediate regimes with non-integer dimensions. This provides new insights to extend critical exponents descriptions into more general or complex scenarios.

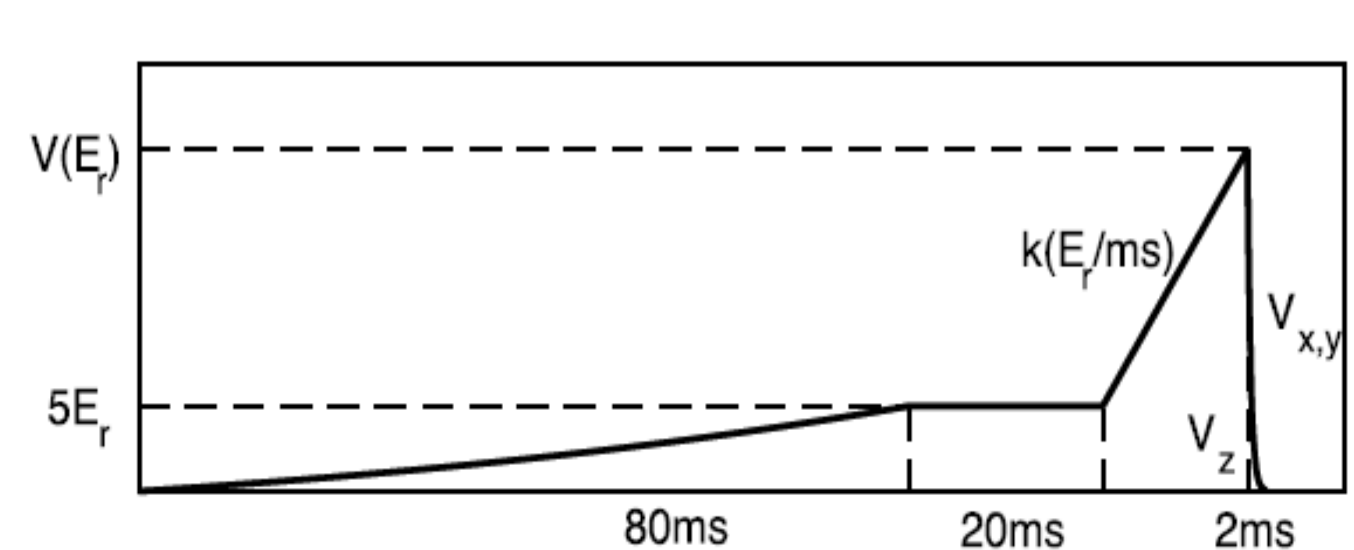
## Improved Band-mapping Method



### Experimental setup and improved band-mapping method:

(a) Three orthogonal standing waves form 3D optical lattices for <sup>87</sup>Rb and absorption images are taken along the z direction. The inset: the z lattice is turned off instantaneously while the x and y lattices are ramped down in 2 ms for the band mapping. This method can distinguish the incoherent part more quantitatively. (b) The incoherent fraction  $\gamma_{inc}$  versus the trap depth  $V$  for the SF-MI phase transitions. The red circles correspond to the measurement with adiabatic ramping, while the blue squares correspond to the measurement with linear ramping at rates  $k = 2.5E_r/ms$ .

## Kibble-Zurek Mechanism



### The time sequence of the trap depth ramping.

The first stage of 80 ms and the second stage of 20 ms prepare superfluid samples from the condensates. The third stage is the linear ramp with a ramping rate  $k$  when the atoms experience phase transitions. The final stage is the band mapping.

- The Kibble-Zurek mechanism (KZM) predicts that:  $\tau \propto k^{-\frac{vz}{1+vz}}$

For  $0.7E_r/ms \leq k \leq 4E_r/ms$ , we have  $\tau_{MI} \propto k^{-0.50(5)}$  which means  $vz = 1.0(2)$

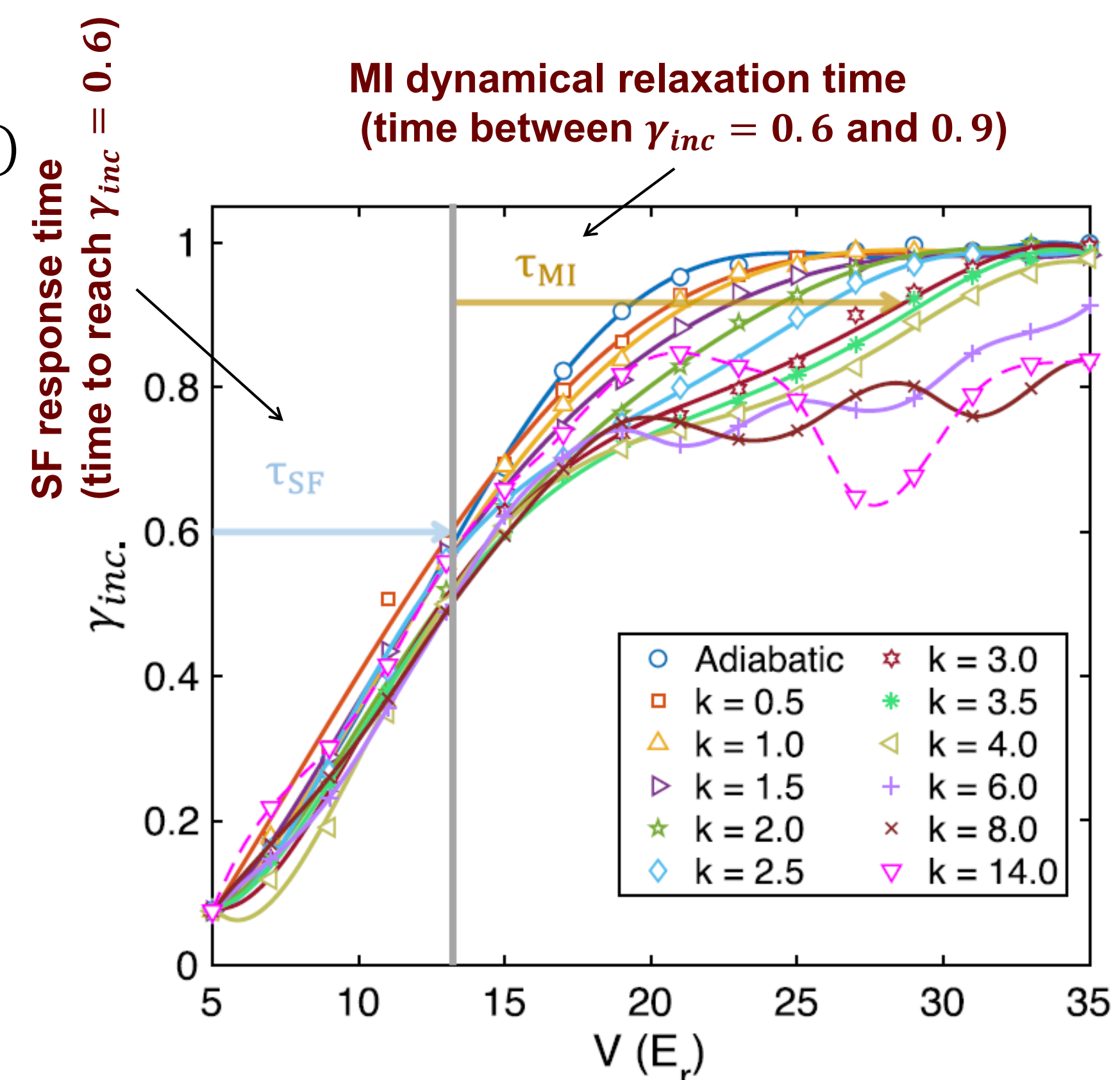
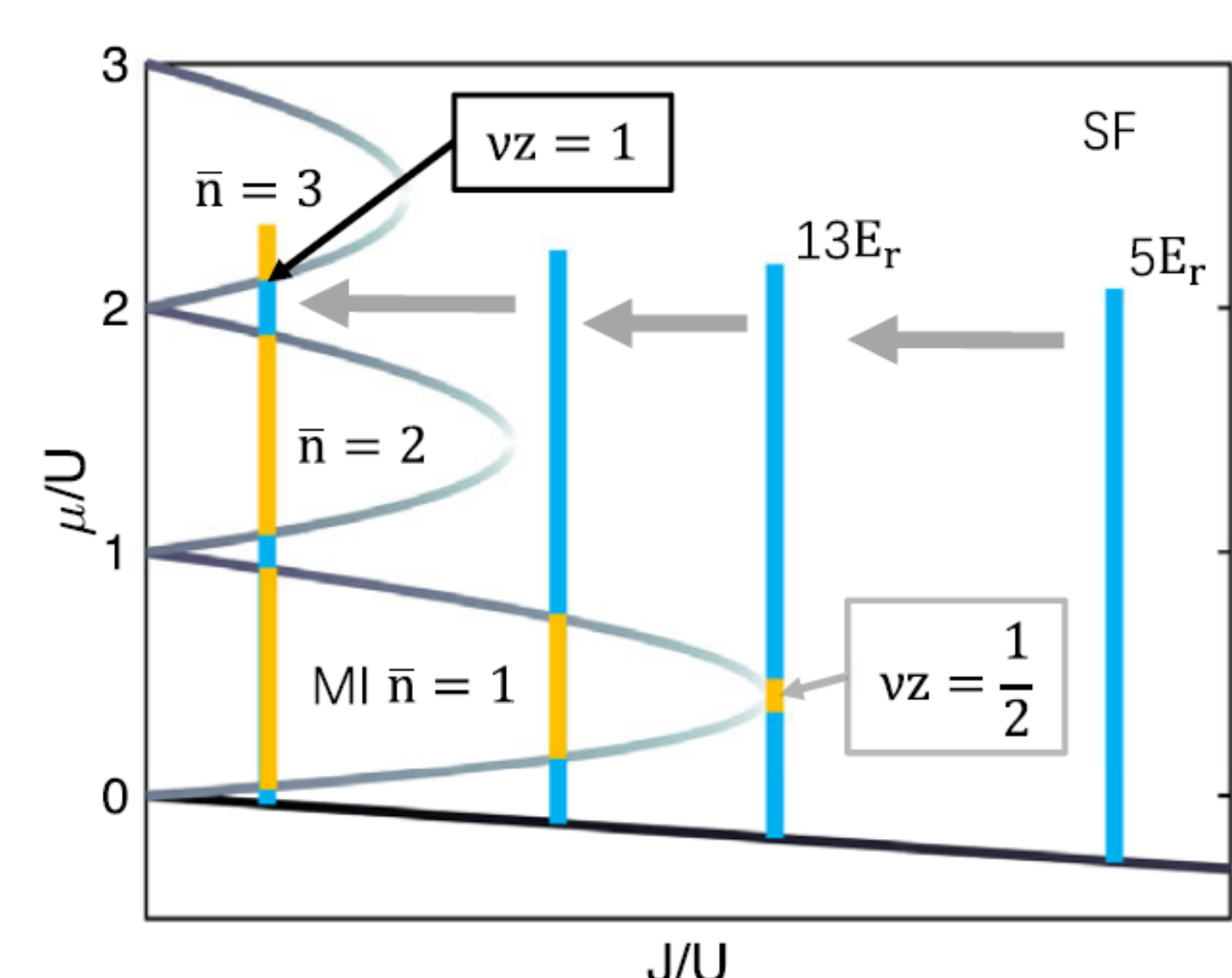
$$v = \frac{1}{2}, z = 2 \rightarrow \text{off-tip region with linear gap}$$

- KZM also predicts that:  $n_{ex} \propto k^{\frac{dv}{1+vz}}$  where  $n_{ex} = \gamma_{inc}(\text{adiabatic}) - \gamma_{inc}(k)$  defines the excitation fraction.

For  $0.7E_r/ms \leq k \leq 4E_r/ms$ , we have  $n_{ex} \propto k^{0.97(12)}$ ,  $vz = 0.48(9)$

$$v = \frac{1}{2}, z = 2$$

on-tip region with square-root gap

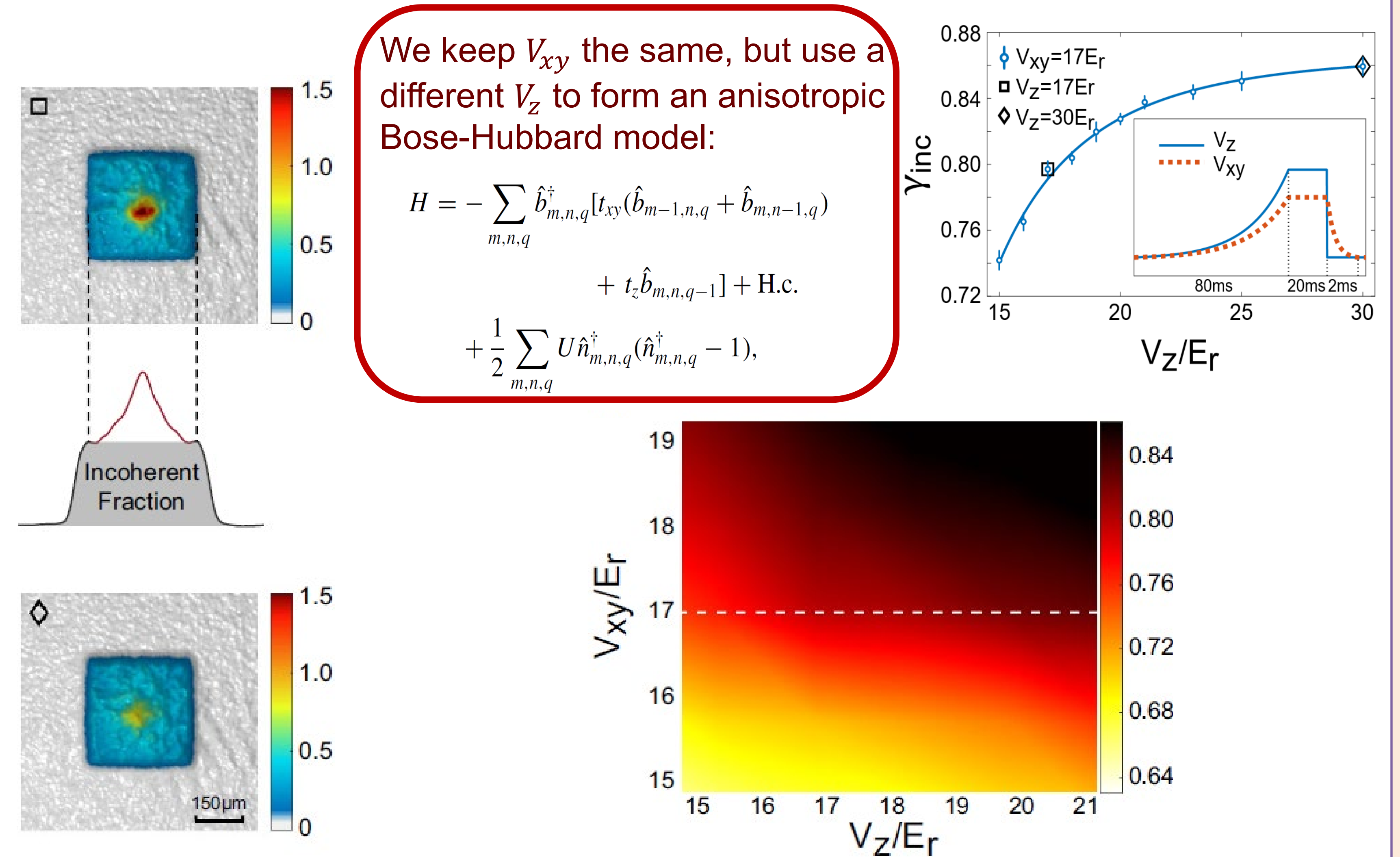


### The incoherent fraction $\gamma_{inc}$ versus trap depth $V$ for different $k$ .

A furcation appears at critical point  $V_c = 13E_r$ . And  $\gamma_{inc}$  approaches 1 for different ramping rate  $k$  for small  $k$ . When  $k$  gets larger,  $\gamma_{inc}$  starts to oscillate with retard thermalization.

## Steady-state Calibration

We prepare the Bose-Einstein condensates with  $1.5(2) \times 10^5$  rubidium-87 atoms with a radius around 10  $\mu\text{m}$  and load them into three-dimensional optical lattices.

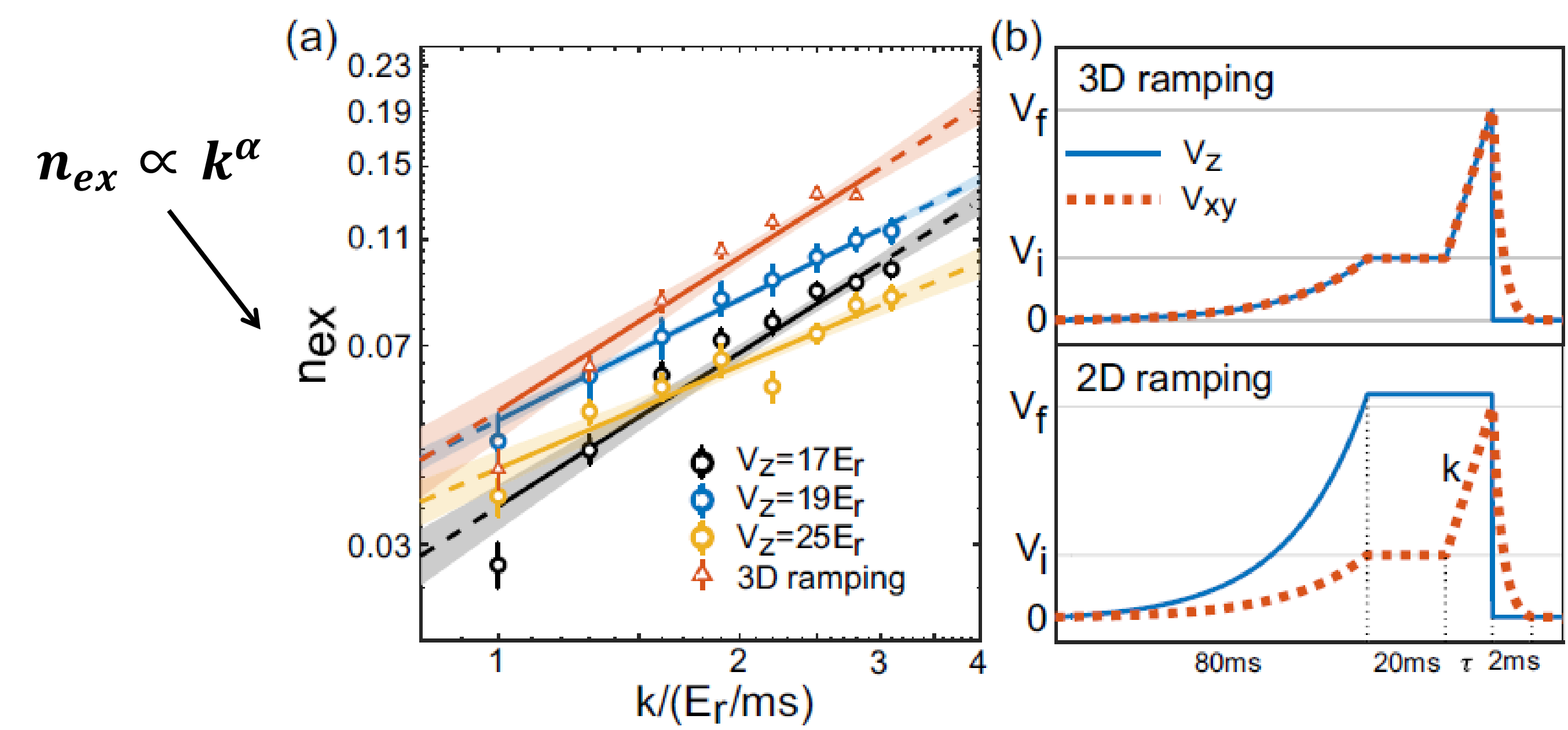


We keep  $V_{xy}$  the same, but use a different  $V_z$  to form an anisotropic Bose-Hubbard model:

$$H = - \sum_{m,n,q} \hat{b}_{m,n,q}^\dagger [t_{xy}(\hat{b}_{m-1,n,q} + \hat{b}_{m,n-1,q}) + t_z \hat{b}_{m,n,q-1}] + \text{H.c.} + \frac{1}{2} \sum_{m,n,q} U \hat{n}_{m,n,q}^\dagger (\hat{n}_{m,n,q} - 1)$$

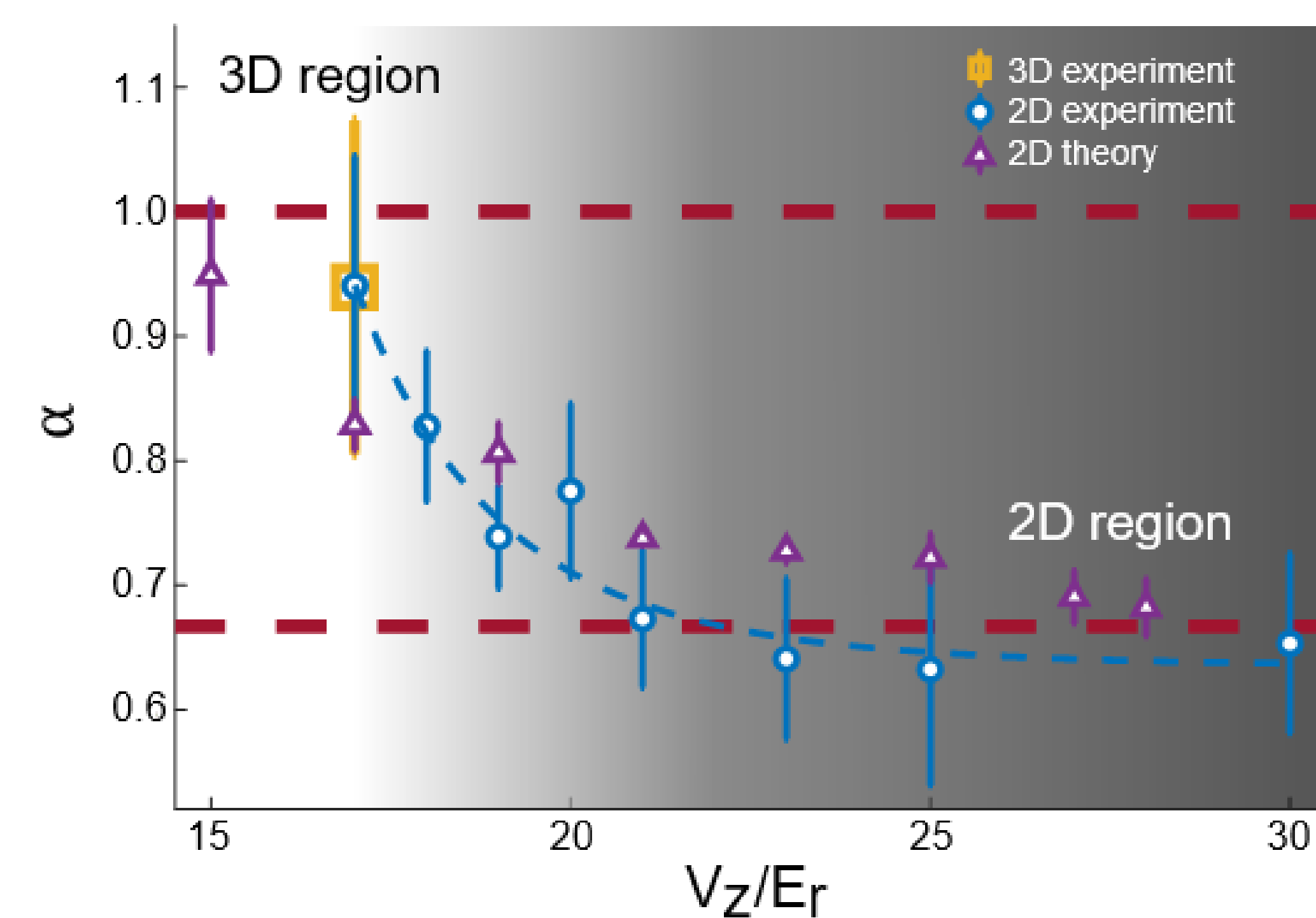
The incoherent fraction  $\gamma_{inc}$  vs the trap depth  $V_{xy}$  and  $V_z$ . Darker color, higher incoherent fraction.

## Dimensional Crossover



- (a) Excitation fraction  $n_{ex}$  vs ramping speed, where  $n_{ex}$  characterizes the topological defects or excitation due to the nonadiabatic ramping:  $n_{ex} = \gamma_{inc}(\text{adiabatic}) - \gamma_{inc}(k)$ . Based on the fit,  $\alpha$  equals **0.94(10)**, **0.74(4)**, and **0.63(9)** for the 2D ramping with  $V_z = 17, 19, \text{ and } 25E_r$ , and  $\alpha$  equals **0.93(11)** for the 3D ramping.
- (b) The ramping protocol of optical lattice depths for the two-dimensional and three-dimensional schemes.

### The critical exponent $\alpha$ vs the trap depth $V_z$ of the third dimension



$$\begin{cases} n_{ex} \propto k^1, d = 3 \\ \text{crossover}, 2 < d < 3 \\ n_{ex} \propto k^{2/3}, d = 2 \end{cases}$$

There appears to be a smooth connection from 1 to 2/3 without any sudden jumps. All the critical exponents are bounded by four identities. Any changes in  $dv$  will result in changes in other critical exponents. This suggests a region with different critical scalings compared with the interger-dimensional systems.

## References

- [1] Qinpei Zheng, Yuqing Wang, Libo Liang, Qi Huang, Shuai Wang, Wei Xiong, Xiaoji Zhou, Wenlan Chen, Xuzong Chen, and Jiazhong Hu, Dimensional crossover of quantum critical dynamics in many-body phase transitions, *Phys. Rev. Res.* **5**, 013136 (2023)
- [2] Qi Huang, Ruixiao Yao, Libo Liang, Shuai Wang, Qinpei Zheng, Dingping Li, Wei Xiong, Xiaoji Zhou, Wenlan Chen, Xuzong Chen, and Jiazhong Hu, Observation of many-body quantum phase transitions beyond the Kibble-Zurek Mechanism, *Phys. Rev. Lett.* **127**, 200601 (2021)