# Measurement of the superfluid fraction of a supersolid by Josephson effect [1]

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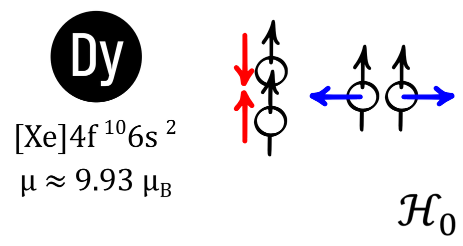
## Supersolids in dipolar quantum gases



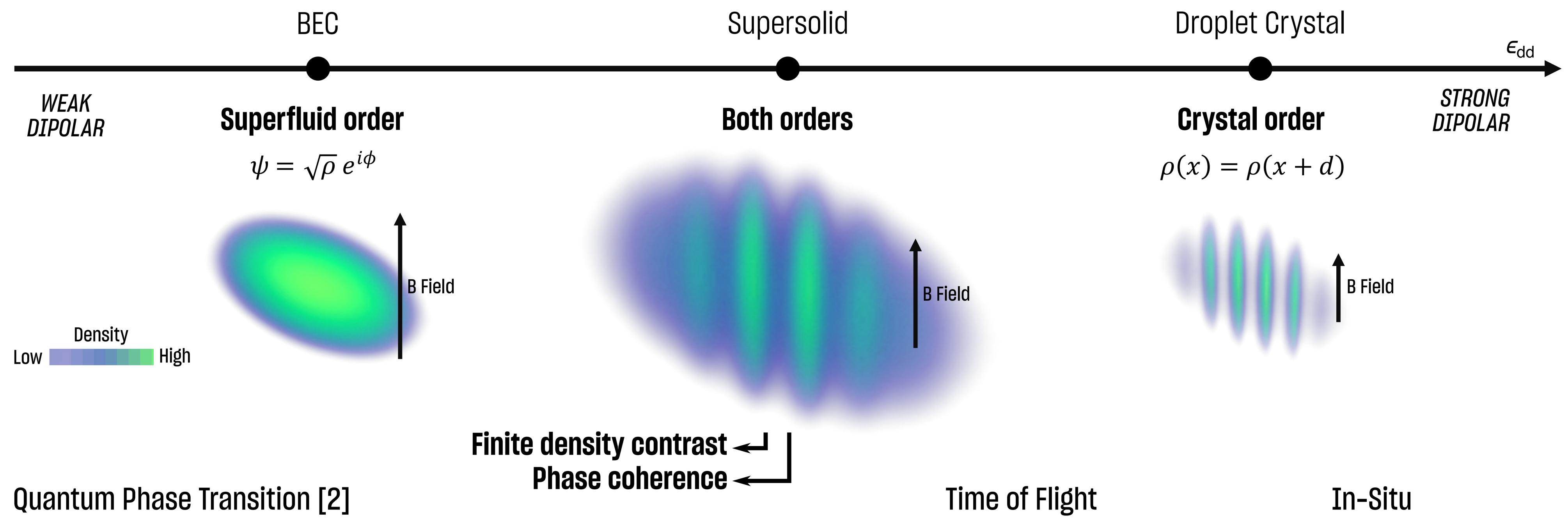

$$U_{dd}(r,\theta) = \frac{\mu_0 \mu^2}{4\pi}\frac{(1-3\cos^2\theta)}{r^3}$$

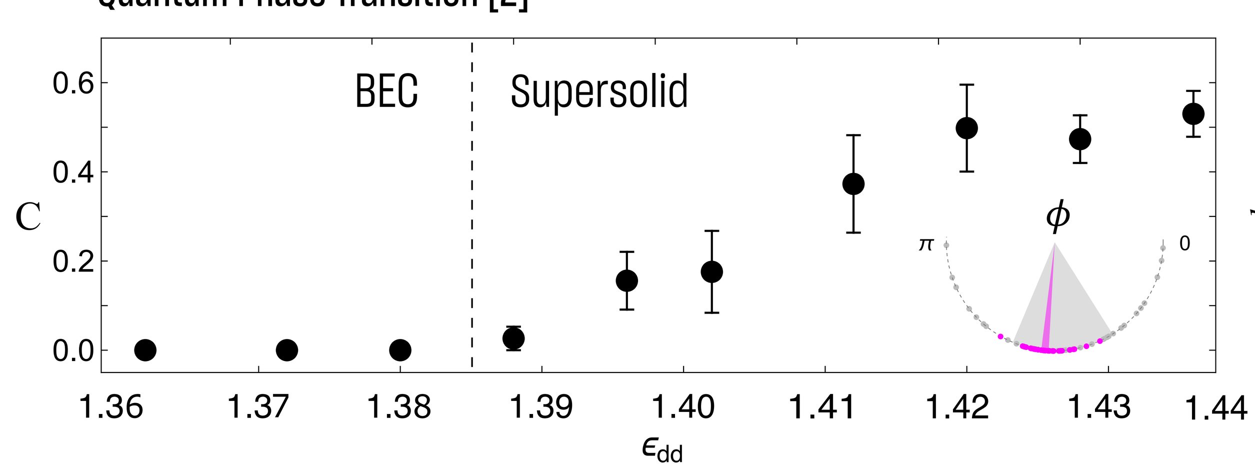
$$H = \underbrace{-\frac{\hbar^2\nabla^2}{2m} + V_{trap}}_{\mathcal{H}_0} + \underbrace{g|\psi|^2}_{\text{contact}} + \underbrace{\int U_{dd}|\psi|^2}_{\text{dipolar}} + \underbrace{H_{LHY}}_{\text{quantum fluctuations}}$$

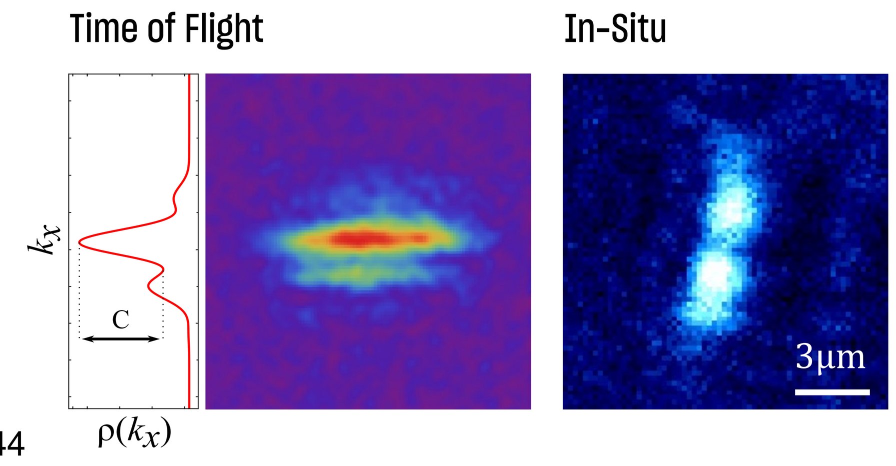
In dipolar quantum gases, atoms can interact via Van der Waals (contact) interactions and **dipole-dipole interactions,** which are **long-range and anisotropic**. The attractive part of the dipolar interaction leads to an instability that clusters atoms together.

A supersolid is a phase of matter where both the **global U(1) symmetry** and **translational symmetry** are broken. Starting from a Bose-Einstein Condensate (superfluid) and changing the strength of the interactions, we cross a **quantum phase transition** towards a system with a crystalline structure without losing the phase coherence.

Dipolar supersolids are of the **cluster type**, since each lattice site contains thousands of atoms. The lattice is **compressible**. Experimental detection is performed after a **long-time expansion** or with **in-situ phase-contrast imaging**.



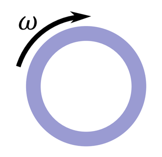




## Superfluid Fraction and Phase Twists

A peculiar property of supersolids is that they **do not behave "fully" like a superfluid, even at T=0,** where the condensed fraction of the system is $\rho_c/\rho \approx 1$.

**Leggett's theory** [3]: reduced moment of inertia under rotations [4]

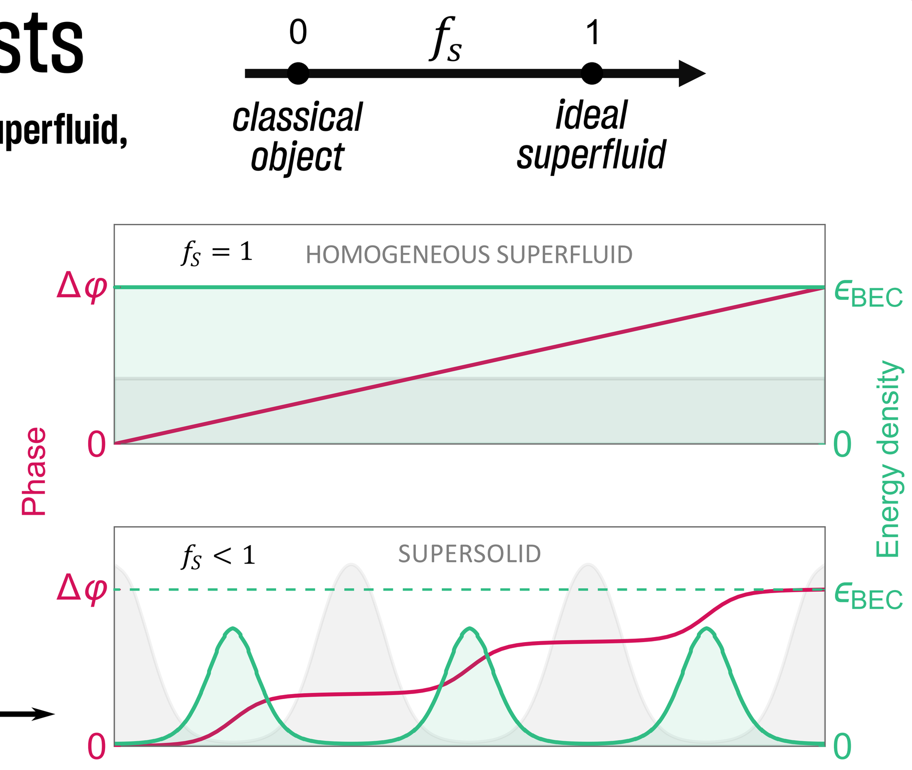


$$I = \left(1 - f_s\right) I_c \qquad f_s^{\,L} = \left(\frac{1}{d}\int_0^d \frac{dx}{\rho(x)/\bar{\rho}}\right)^{-1}$$

**General formulation**: kinetic energy cost from a phase twist

$$\Delta E_{kin} = \frac{\hbar^2}{2m}\int d\mathbf{r}\,\rho(\mathbf{r})|\nabla\phi(\mathbf{r})|^2 \qquad f_s = \frac{\Delta E_{kin}}{(\Delta E_{kin})_{SF}}$$

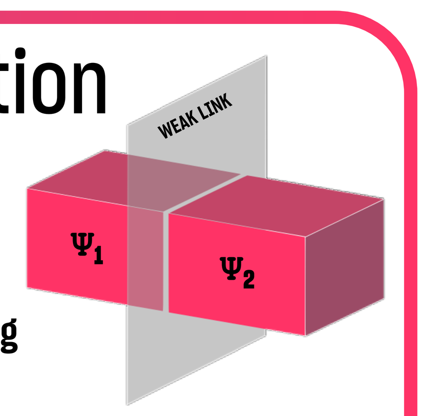
Rotations are mapped as global phase twists



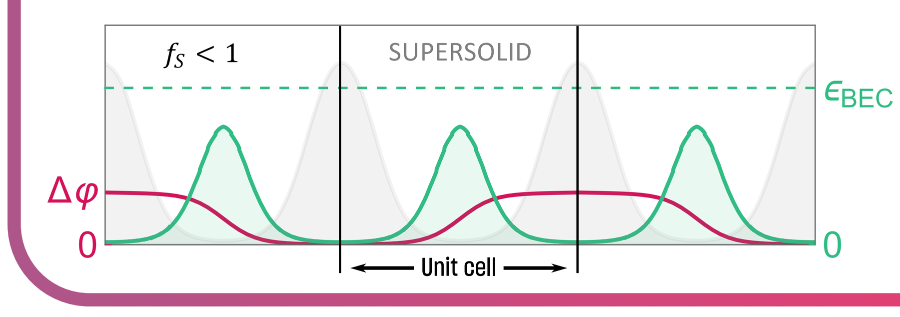

## Self-Induced Josephson Junction

The supersolid density modulation **spontaneously** create **weak links** connecting adjacent superfluid reservoirs, realizing an array of Josephson junctions.

The supercurrent across each junction is regulated by the **tunneling energy K**, which is proportional to the superfluid fraction:




$$\Delta E_{kin} = NK\Delta\phi^2 \qquad f_s = \frac{K}{\hbar^2/(2md^2)}$$



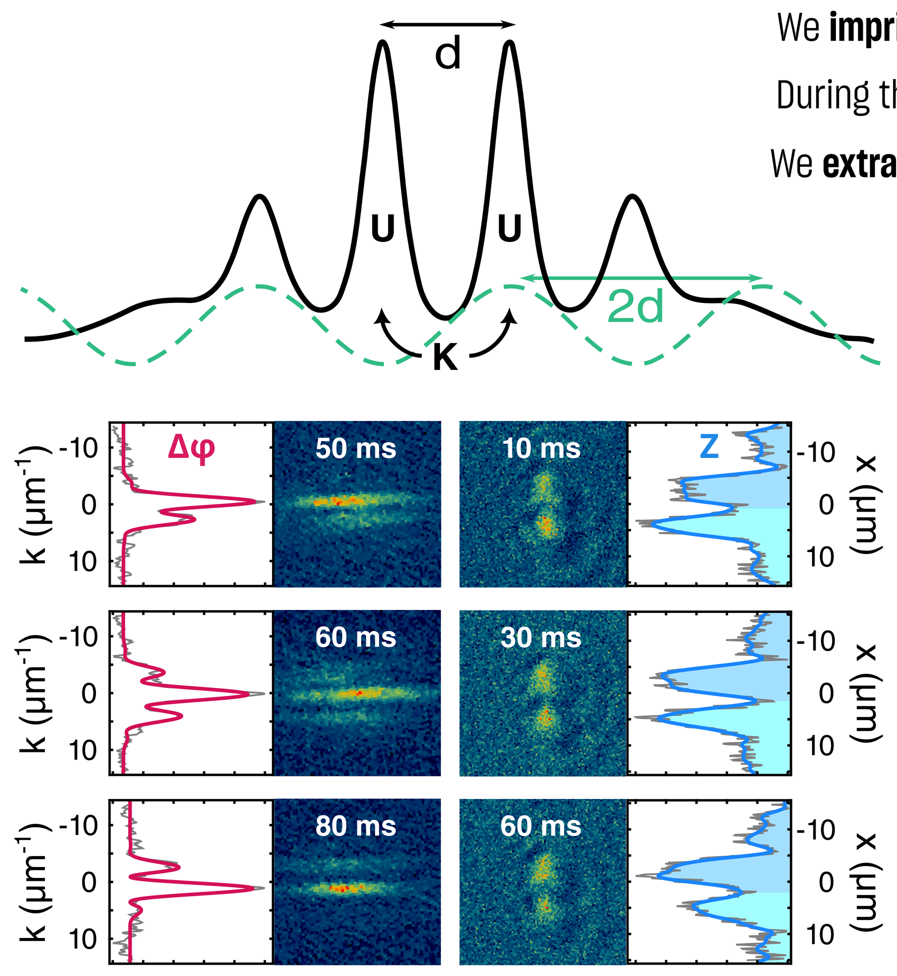

Imprinting a **phase twist with odd parity** with respect to the unit cell of the supersolid would trigger **Josephson oscillations.**

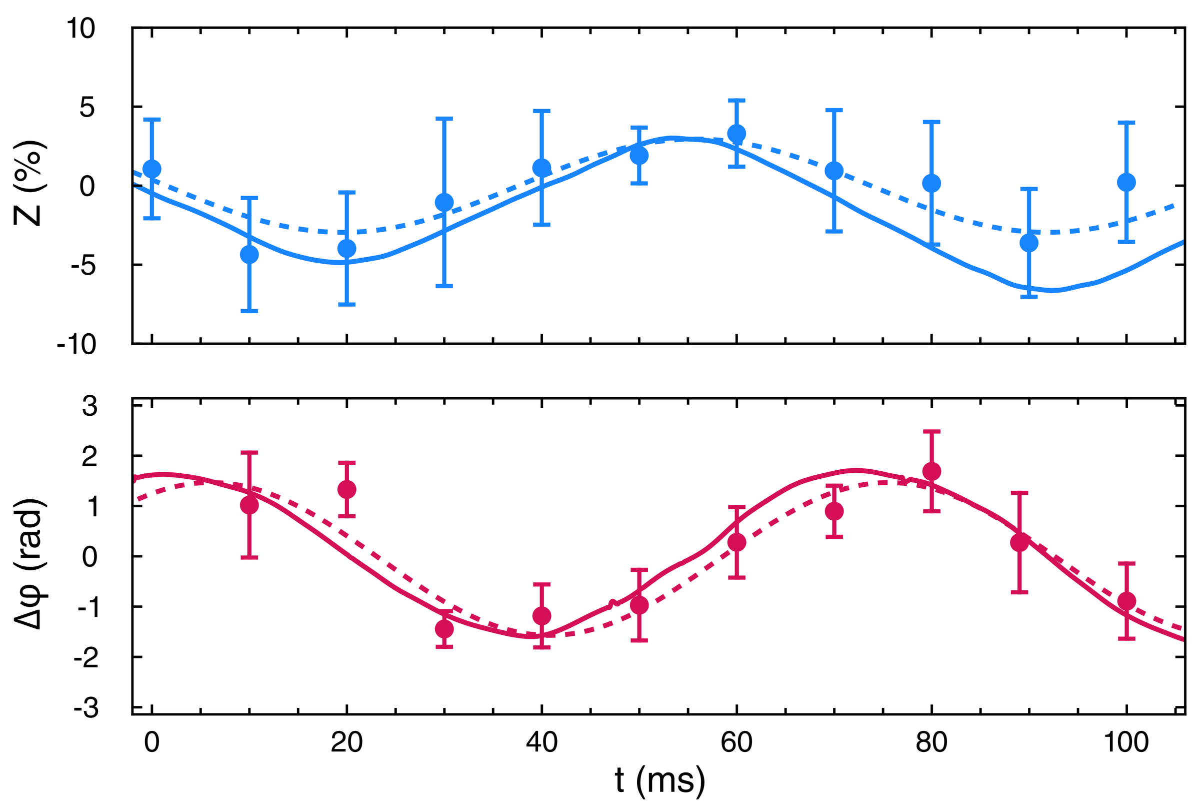
## Observation of Josephson Oscillations in a Supersolid

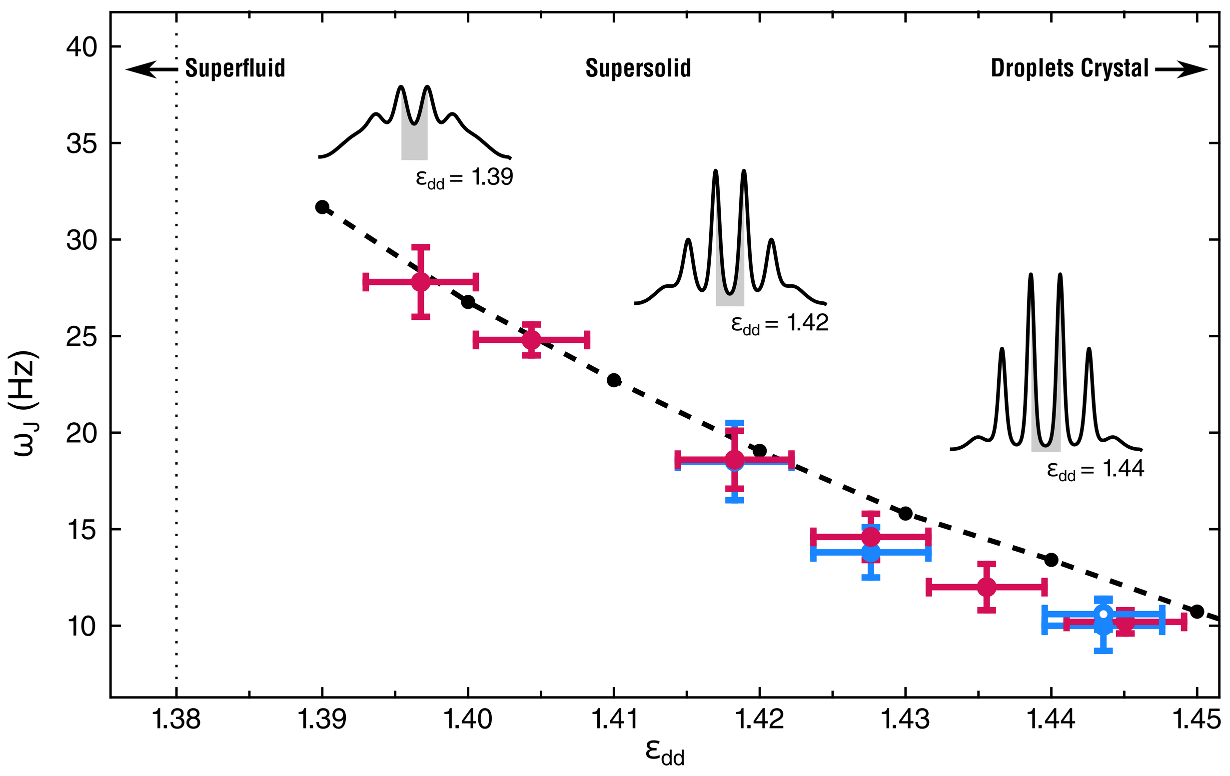
We **imprint a phase difference** between the two principal clusters, shining a **large-spaced optical lattice** for a very brief time interval.

During the Josephson dynamics the relative phase $\phi$ and the population imbalance $z$ can be detected using standard ToF absorption imaging or in-situ phase contrast imaging.

We **extract Josephson frequencies** from both $z$ and $\phi$, and compare the results with numerical simulations of the so called **extended Gross-Pitaevskij Equations.**

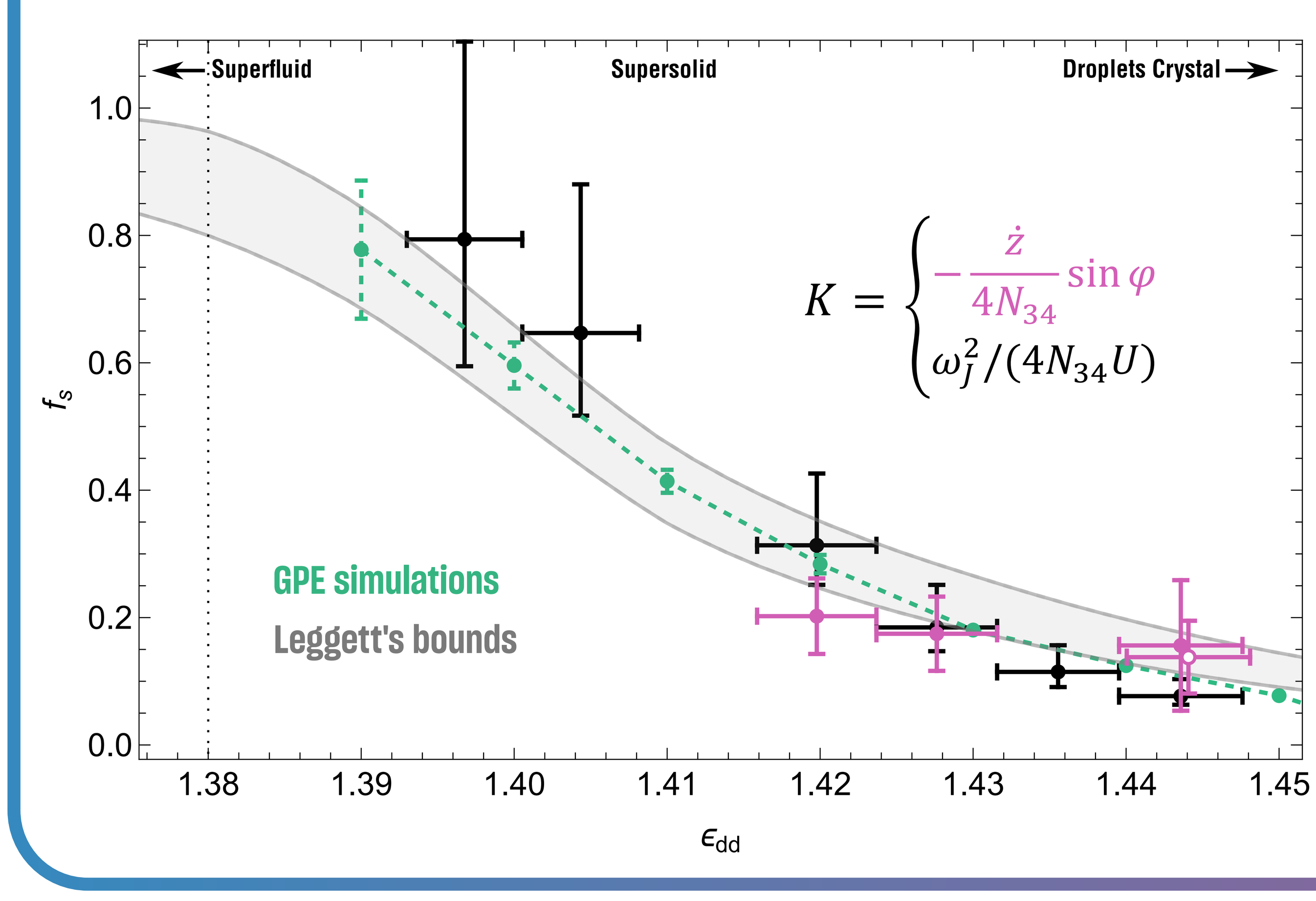


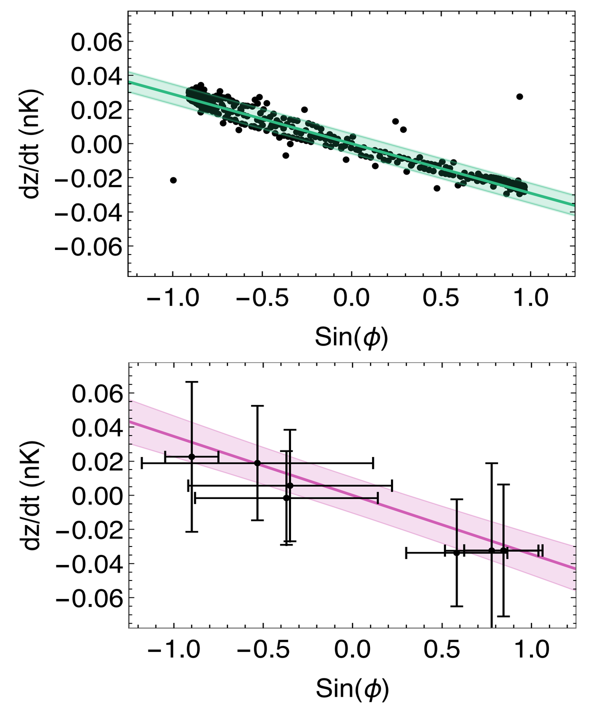

## Measuring the Sub-Unity Superfluid Fraction

The superfluid fraction of the system **can be extracted from data** of Josephson oscillations: from the current-phase relations (pink points), or from the frequency $\omega_J$ of Josephson oscillations.




$$K = \begin{cases} -\dfrac{\dot{z}}{4N_{34}}\sin\varphi \\ \omega_J^2/(4N_{34}U) \end{cases}$$

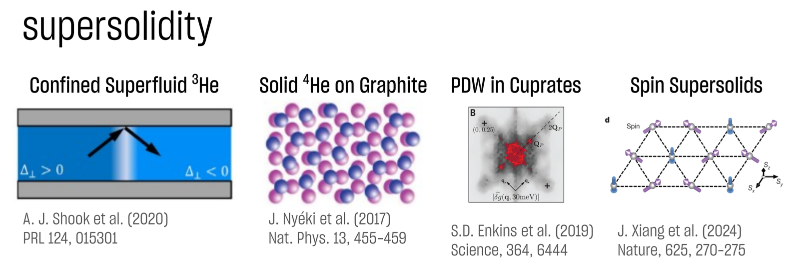
$$f_s = \frac{K}{\hbar^2/(2md^2)}$$



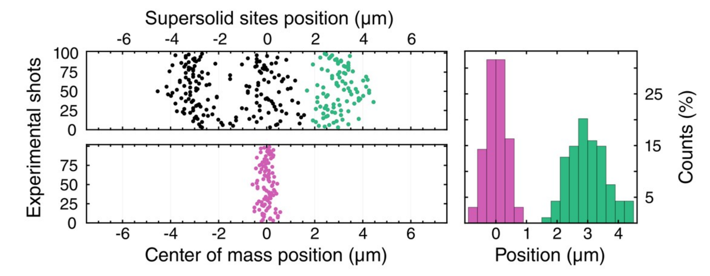

## Conclusions and Outlook

Dipolar supersolids realize a **new type of Josephson junction** without externally imposed barriers. **The Josephson current is a direct probe of the superfluid fraction**. We assess a superfluid fraction well below unit and tunable across the whole supersolid regime.

The concept of a reduced superfluid fraction due to a competing crystal-like order could be **extended to many new condensed matter systems**, establishing or strengthening their connection to supersolidity




The movement of the barrier (**Goldstone mode**) can play an important role in the system **fluctuations**: new entaglement properties of the self-induced Josephson junction?




Exhotic phenomena due to the sub-unity superfluid fraction are expected: **partially quantized [5] and counter-propagating persistent currents in ring geometries.**

## References

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