Generating scalable graph states in an atom-nanophotonic interface

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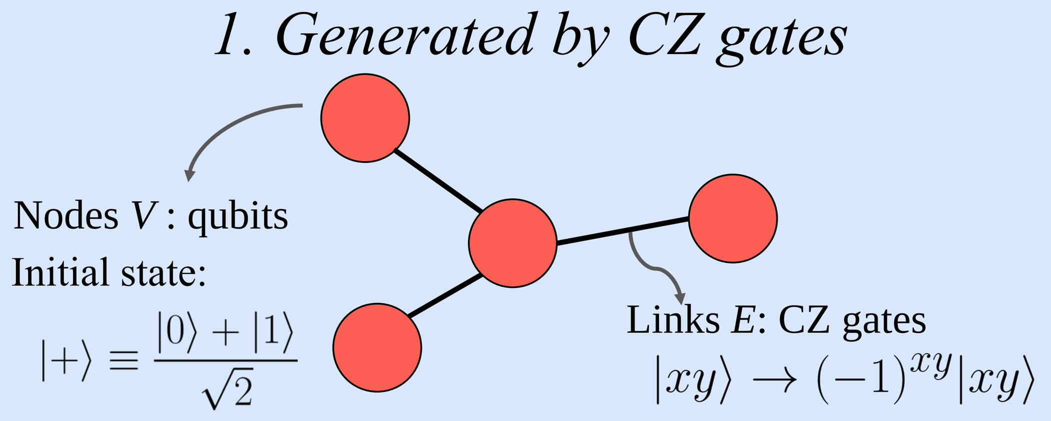
I. Graph state (GS)[1]

It is a kind of maximally entangled state associated with a graph $G(V, E)$

1. Generated by CZ gates



2. Stabilizer code

$$K_G|G\rangle = |G\rangle$$

$$K_G^{(a)} = \sigma_x^{(a)} \bigotimes_{b \in N(a)} \sigma_z^{(b)}$$

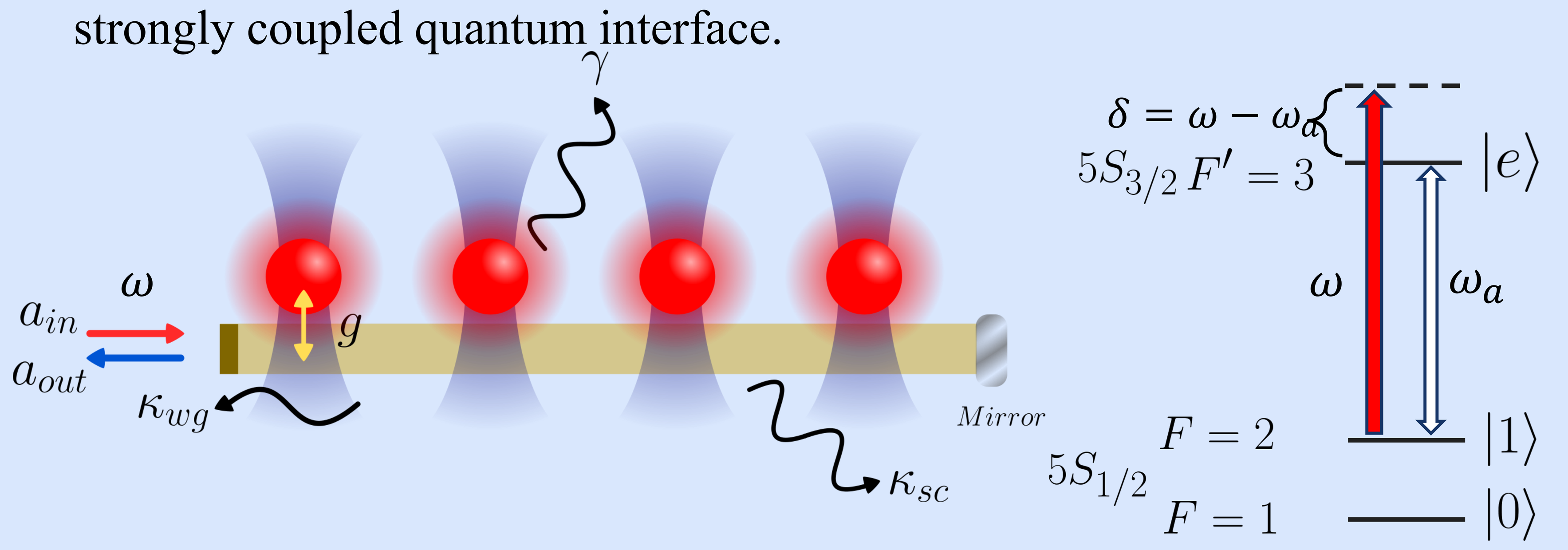
$N(a)$: Neighborhood of a

Instead of using CZ gate (controlled-Z gate), we tried to find the protocols to generated atomic graph states by **state carving**, which generates entanglement by simply sending and detecting photons.

II. State carving[2-4]

1. Atom-nanophotonic interface

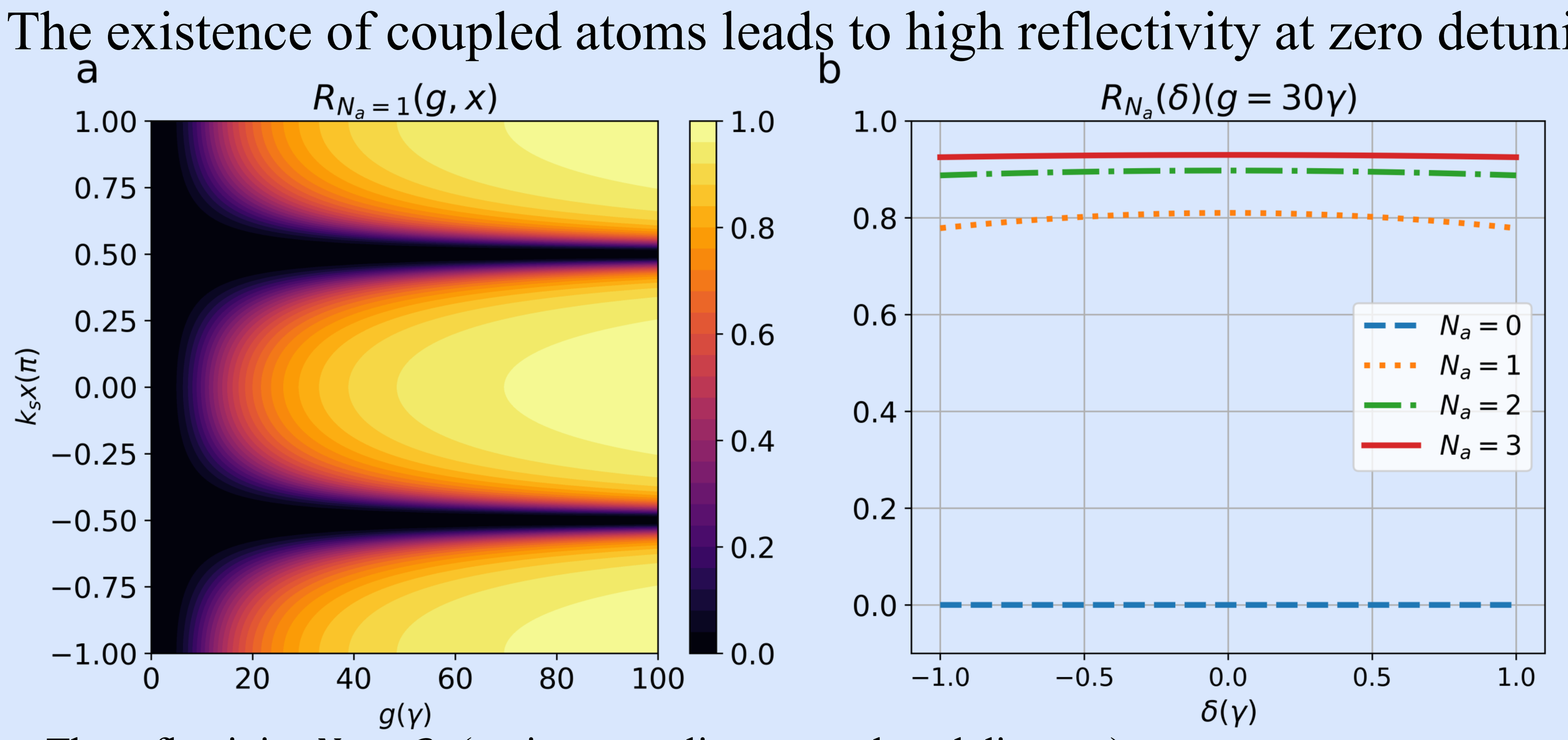
An optical tweezer array of atoms trapped near the waveguide presents a strongly coupled quantum interface.



g: Coupling strength
γ: The decay rate to the free space
N_a: Number of coupled atoms
κ_{wg}: The cavity decay rate to the waveguide
κ_{sc}: The loss rate for scattered photons
ω: The frequency of the input photon
ω_a: The frequency of atomic transition

2. Single photon reflectivity

The existence of coupled atoms leads to high reflectivity at zero detuning



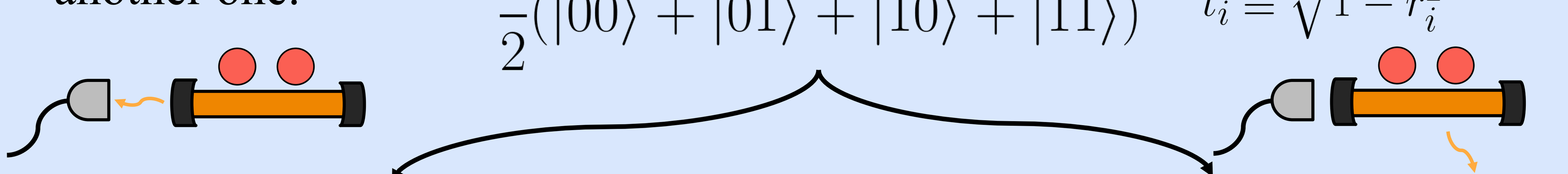
a. The reflectivity $N_a = 2$. (various coupling strength and distance)
b. The reflectivity at the **critical coupling regime** $\kappa_{wg} \approx \kappa_{sc}$. (Distances = integer · π)

The on-resonance reflectivity with $N_a = 0$ drops to zero under the **critical coupling regime**. This huge contrast of single-photon reflectivity enable us to generate heralded entanglement.

3. Heralded entanglement

With the contrast reflectivity, photon detection projects the state onto another one.

$$\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \qquad t_i \equiv \sqrt{1 - r_i^2}$$

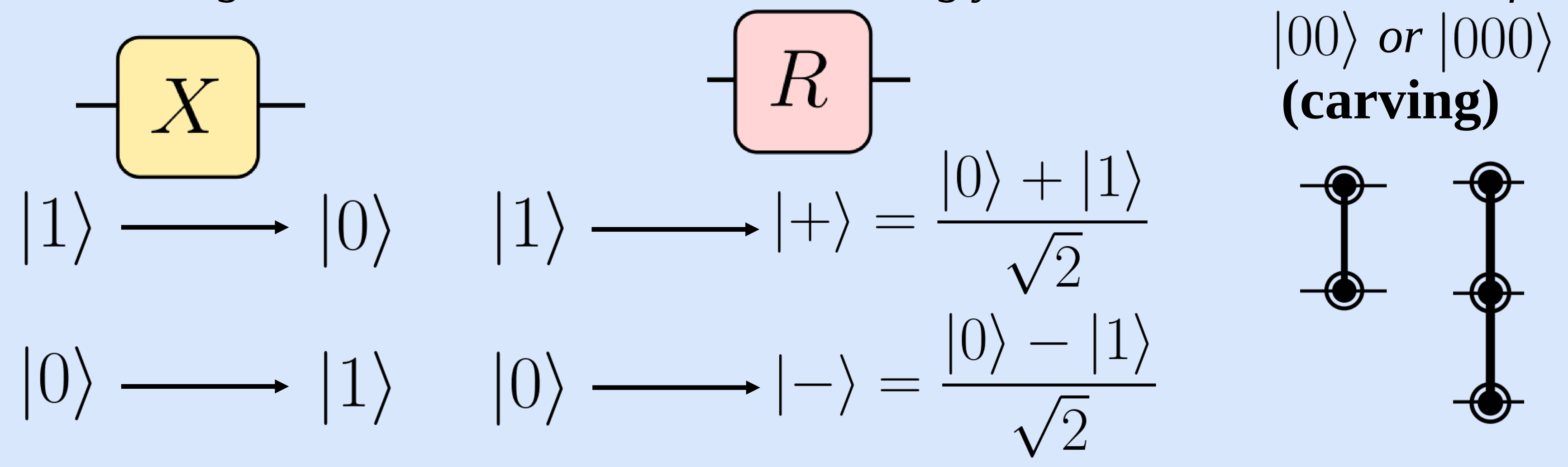


$$\frac{1}{C}(r_0|00\rangle + r_1|01\rangle + r_1|10\rangle + r_2|11\rangle) \qquad \frac{1}{C}(t_0|00\rangle + t_1|01\rangle + t_1|10\rangle + t_2|11\rangle)$$

III. Protocols

Notations

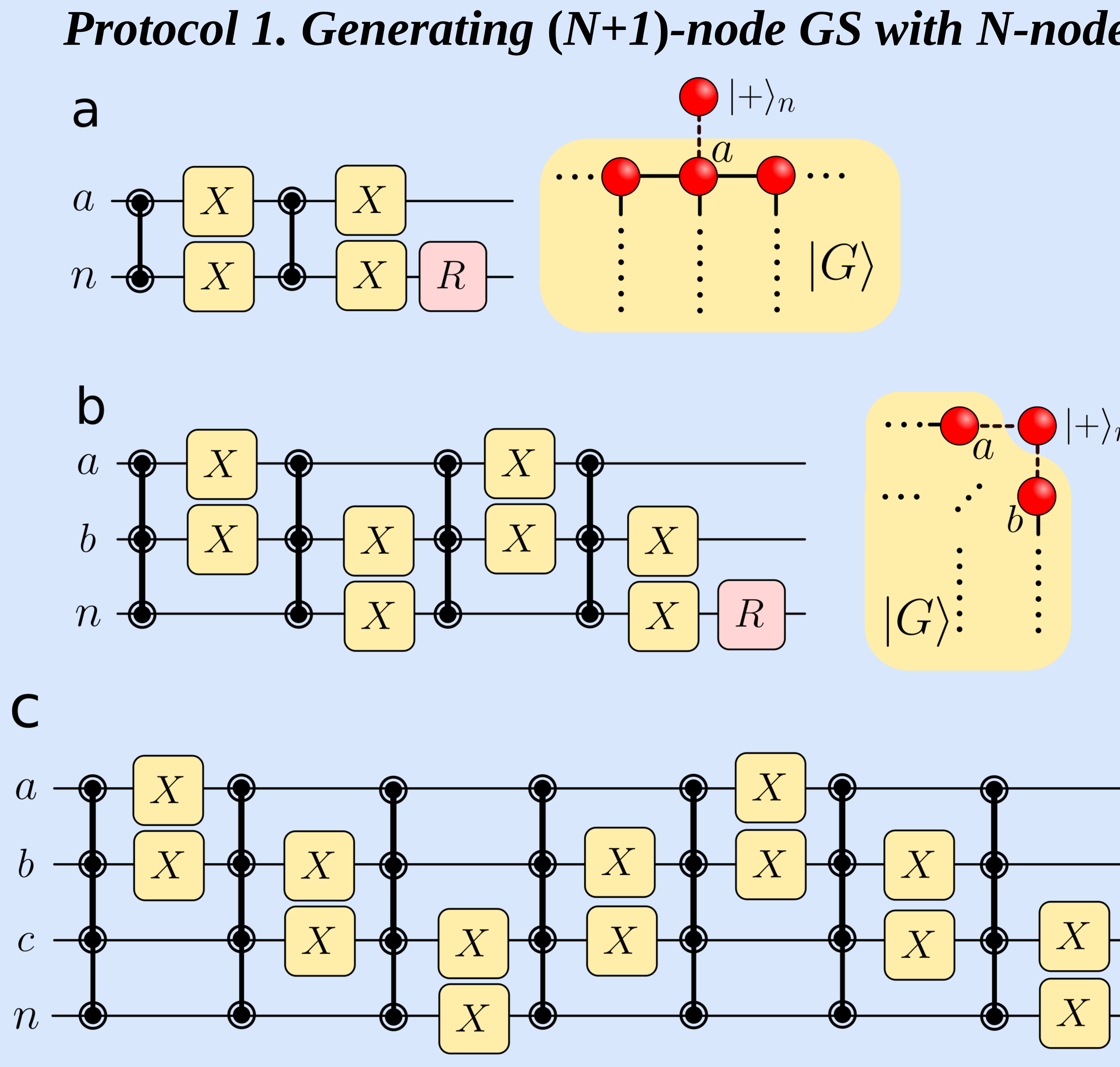
1.Pauli-X gate — *2.π/2 rotation along y-axis* — *3.Removal of $|00\rangle$ or $|000\rangle$* **(carving)**



$|1\rangle \longrightarrow |0\rangle$, $|0\rangle \longrightarrow |1\rangle$

$|1\rangle \longrightarrow |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $|0\rangle \longrightarrow |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

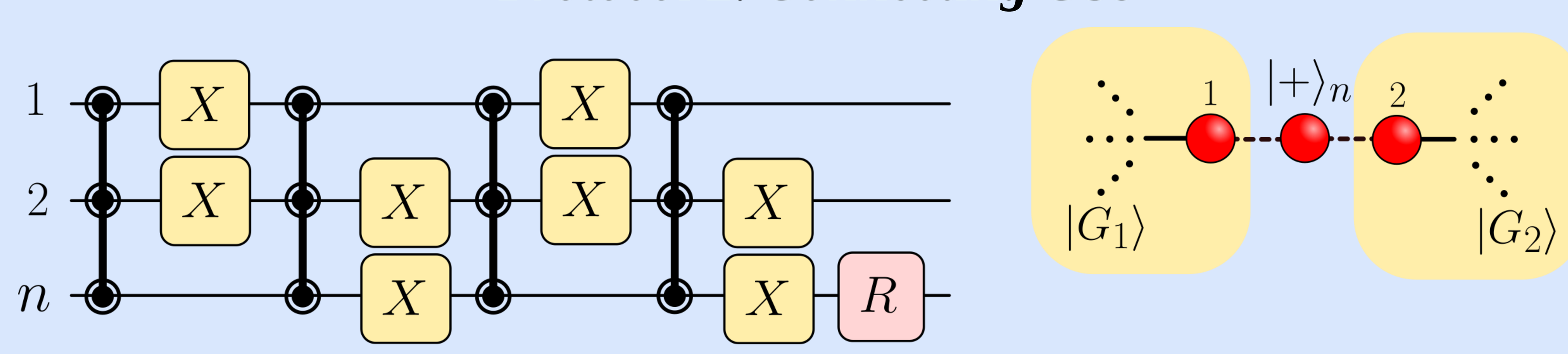
Protocol 1. Generating (N+1)-node GS with N-node GS and an individual qubit



Generating 2-atom GS

Initial state:
$|++\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$
Remove $|00\rangle$: $\frac{1}{\sqrt{3}}(|01\rangle + |10\rangle + |11\rangle)$
X gates: $\frac{1}{\sqrt{3}}(|10\rangle + |01\rangle + |00\rangle)$
Remove $|00\rangle$: $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$
Qubit rotation:
$\frac{1}{\sqrt{2}}(|1+\rangle + |0-\rangle)$

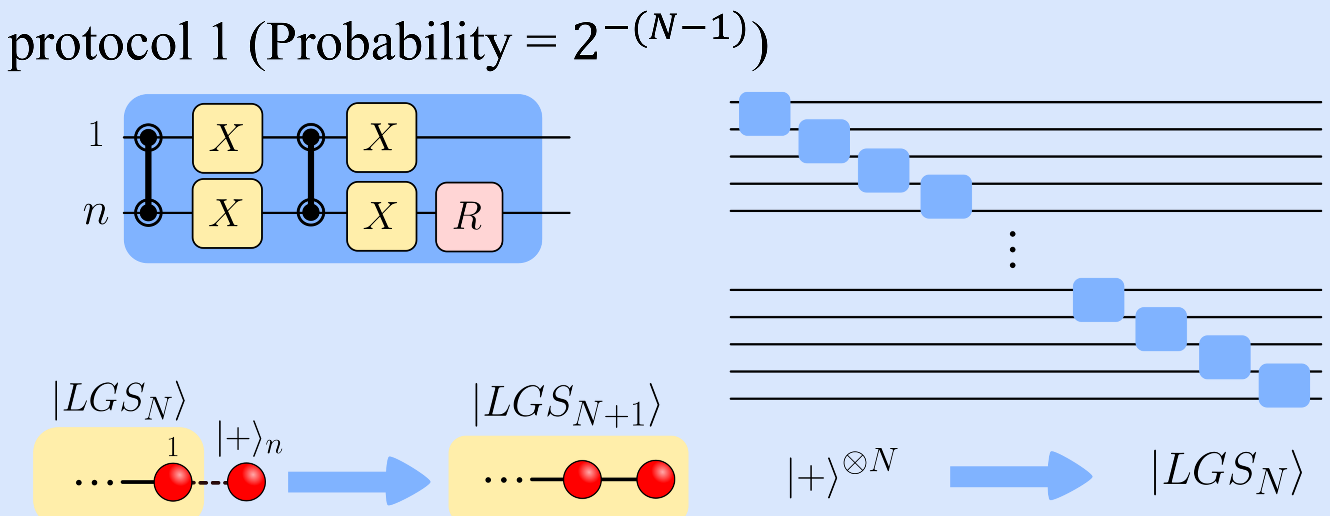
Protocol 2. Connecting GSs



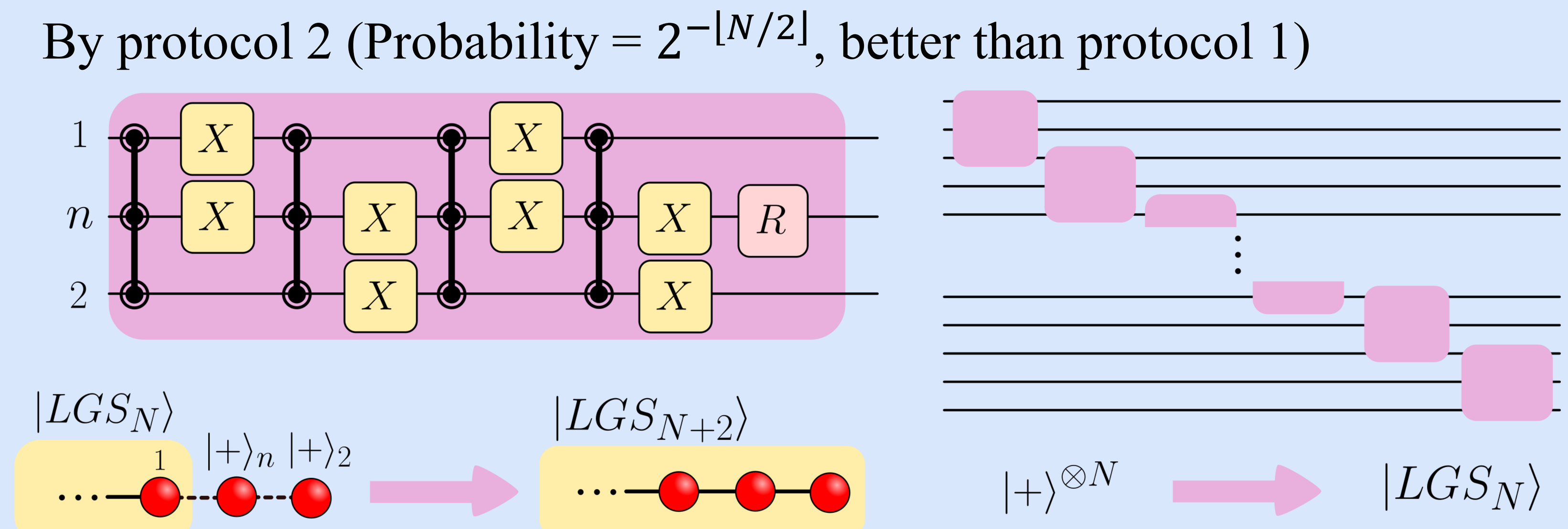
The two protocols can be used to generate an atomic GS of any shape.

IV. Generating linear graph states (LGSs)

- By protocol 1 (Probability = $2^{-(N-1)}$)



- By protocol 2 (Probability = $2^{-\lfloor N/2 \rfloor}$, better than protocol 1)



References

[1] Hein, M., Eisert, J., & Briegel, H. J. (2004). **Physical Review A**, *69*(6).
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[4] Tamara Dordevi´c, Polnop Samutpraphoot, Paloma L. Ocola, Hannes Bernien, Brandon Grinkemeyer, Ivana Dimitrova, Vladan Vuleti´c, and Mikhail D. Lukin. **Science**, *373*(6562):1511–1514, (2021).