Multicritical dissipative phase transitions in the anisotropic open quantum Rabi model

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Abstract

We study a generalized open quantum Rabi model with varying coupling strengths in the rotating and counter-rotating terms. Using both semiclassical and quantum approaches, we find a rich phase diagram resulting from the interplay between the anisotropic interaction and the dissipation.

First, there exists a bistable phase where both the normal and superradiant phases are stable. Second, there are multicritical points where the phase boundaries for the first- and second-order phase transitions meet. We show that a new set of critical exponents governs the scaling of the multicritical points, which could be utilized for developing critical sensing protocols.

Model: anisotropic open QRM

$$H = \omega_0 a^\dagger a + \tfrac{\Omega}{2}\sigma_z - \lambda_r(a\sigma_+ + a^\dagger\sigma_-) - \lambda_{cr}(a\sigma_- + a^\dagger\sigma_+)$$

$$\dot\rho = -i[H,\rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

Where the anisotropic coupling strengths are denoted by λ_r and λ_{cr} for the rotating and counterrotating terms, and κ is the damping rate of the harmonic oscillator.

We consider that $\Omega/\omega_0 \to \infty$, which serves as the thermodynamical limit. In this limit, the (open) Rabi model exhibits a (dissipative) superradiant phase transition, as predicted in PRL2015 and PRA2018 by Hwang *et.al*.

Semiclassical analysis

- We obtain a mean-field solution for the steady states by solving the semiclassical equations of motion:

$$\langle \dot a\rangle = -i(\omega_0 - i\kappa)\langle a\rangle + i(\lambda_{cr}\langle\sigma_+\rangle + \lambda_r\langle\sigma_-\rangle),$$
$$\langle\dot\sigma_+\rangle = i\Omega\langle\sigma_+\rangle + i(\lambda_{cr}\langle a\rangle + \lambda_r\langle a^\dagger\rangle)\langle\sigma_z\rangle,$$
$$\langle\dot\sigma_z\rangle = i2(\lambda_r\langle a\rangle + \lambda_{cr}\langle a^\dagger\rangle)\langle\sigma_+\rangle + \text{c.c.}$$

$\langle\dot a\rangle \equiv \frac{d}{dt}\langle a\rangle$, $\langle a\rangle \equiv \mathrm{Tr}[\rho a]$

The solution for the **normal phase (NP):**

$$s_z = 1,\ x = y = s_x = s_y = 0$$

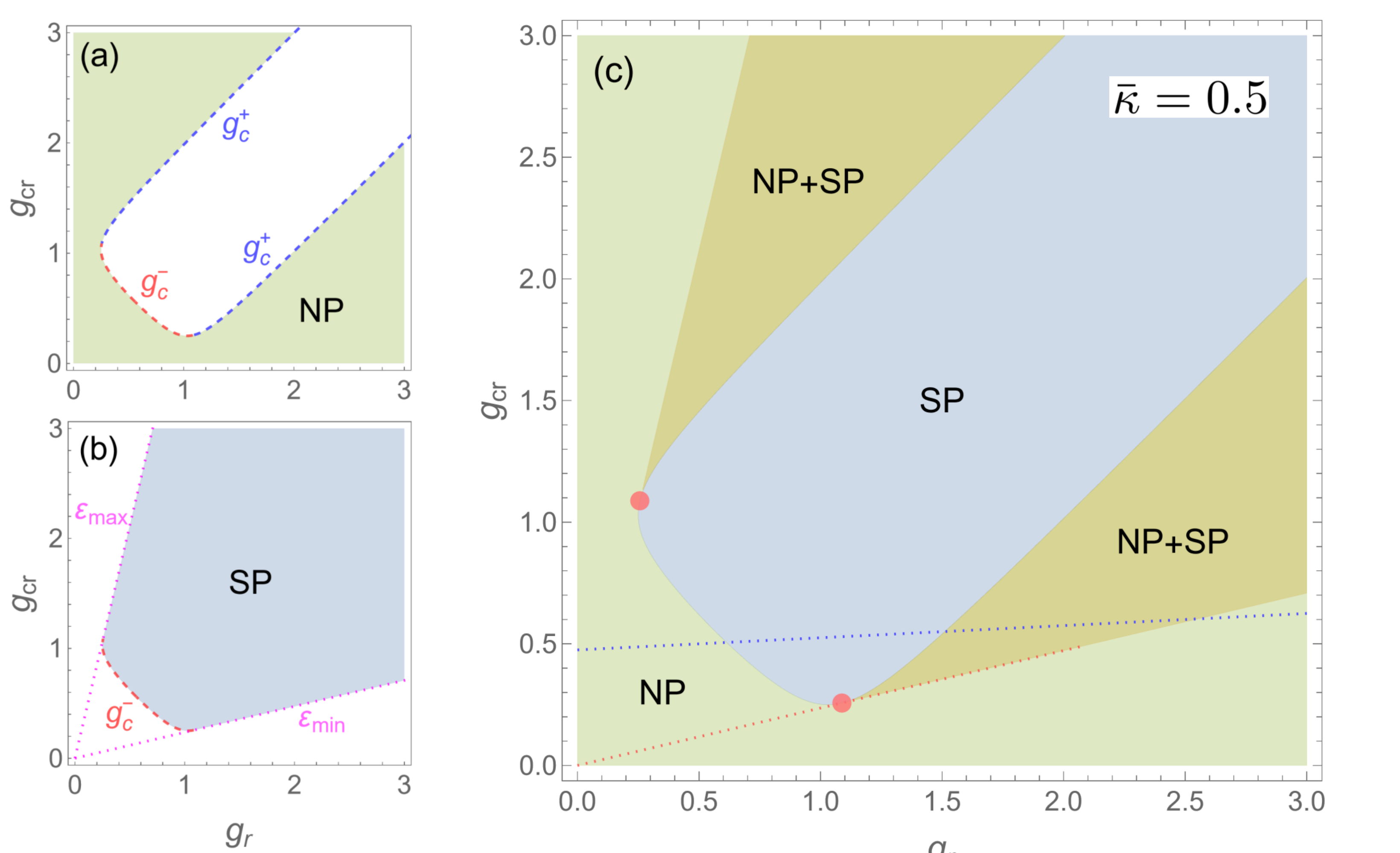
$s_{x,y,z} \equiv \langle\sigma_{x,y,z}\rangle$,

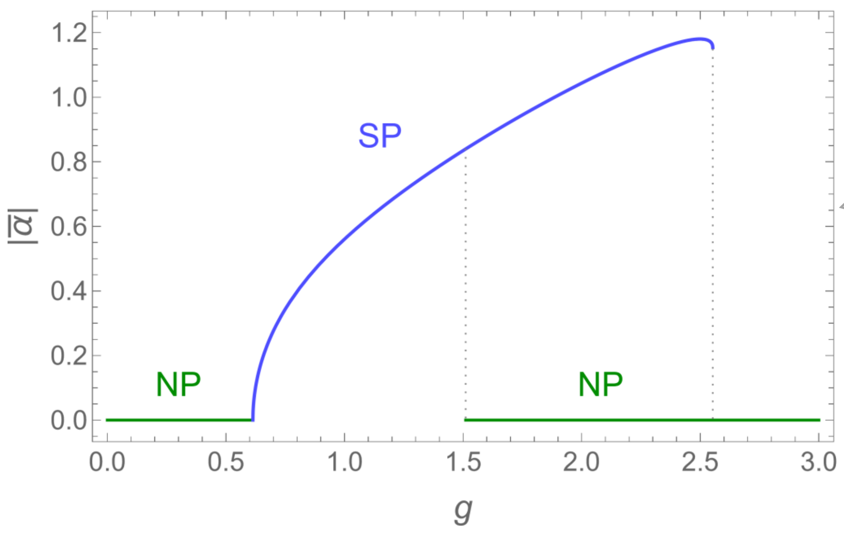
The solution for the **superradiant phase (SP):**

$$s_z = -\frac{(1+\varepsilon^2) \pm \sqrt{4\varepsilon^2 - \bar\kappa^2(1-\varepsilon^2)^2}}{(1-\varepsilon^2)^2 g^2}$$

$g_r \equiv \lambda_r/\sqrt{\omega_0\Omega}$, $g_{cr} \equiv \lambda_{cr}/\sqrt{\omega_0\Omega}$, $\bar\kappa \equiv \kappa/\omega_0$, $g_r \equiv g$, $g_{cr} \equiv \varepsilon g$

- After a stability analysis for above mean-field solutions, we obtain the phase diagram of the system. (The following quantum solutions agree with the phase diagram.)





Dissipative phase transitions. The absolute value of α as gr going along the blue dotted line in Fig. 1(c).

Quantum fluctuations

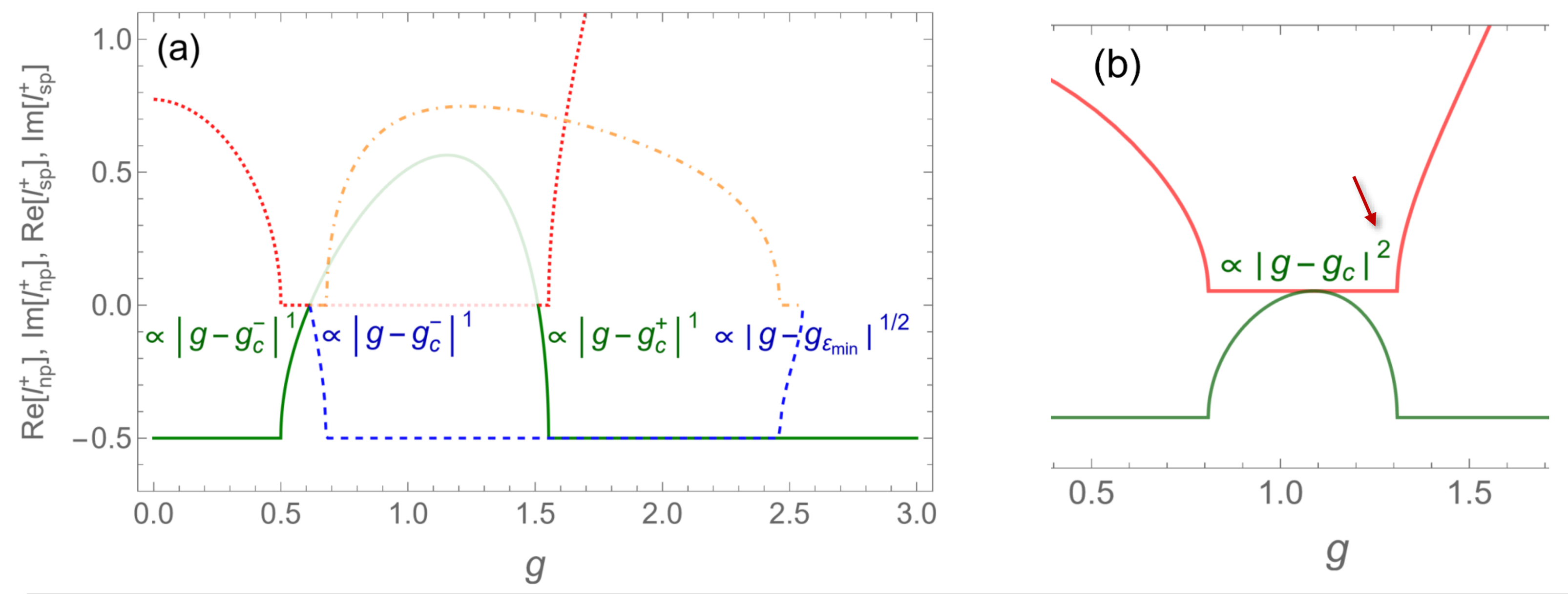
By performing the Schrieffer-Wolff (SW) transformation to the Hamiltonian and projecting it to the spin-down subspace, we obtain the effective Hamiltonian for the NP. (Similarly for the SP, but a displacement to the master equation is necessary.)

$$H_{\rm np} = \omega_0 a^\dagger a - \omega_0\left[g_r^2 a^\dagger a + g_{cr}^2 a a^\dagger + g_r g_{cr}(a^{\dagger 2} + a^2)\right]$$

- First moment of the oscillator operators:

$$\dot{\mathbf a} = L_{\rm np}\mathbf a \qquad \mathbf a = (\langle a\rangle, \langle a^\dagger\rangle)^\intercal$$

$$l_{\rm np}^\pm = -\bar\kappa \pm \sqrt{2g^2(1+\varepsilon^2) - g^4(1-\varepsilon^2)^2 - 1}$$

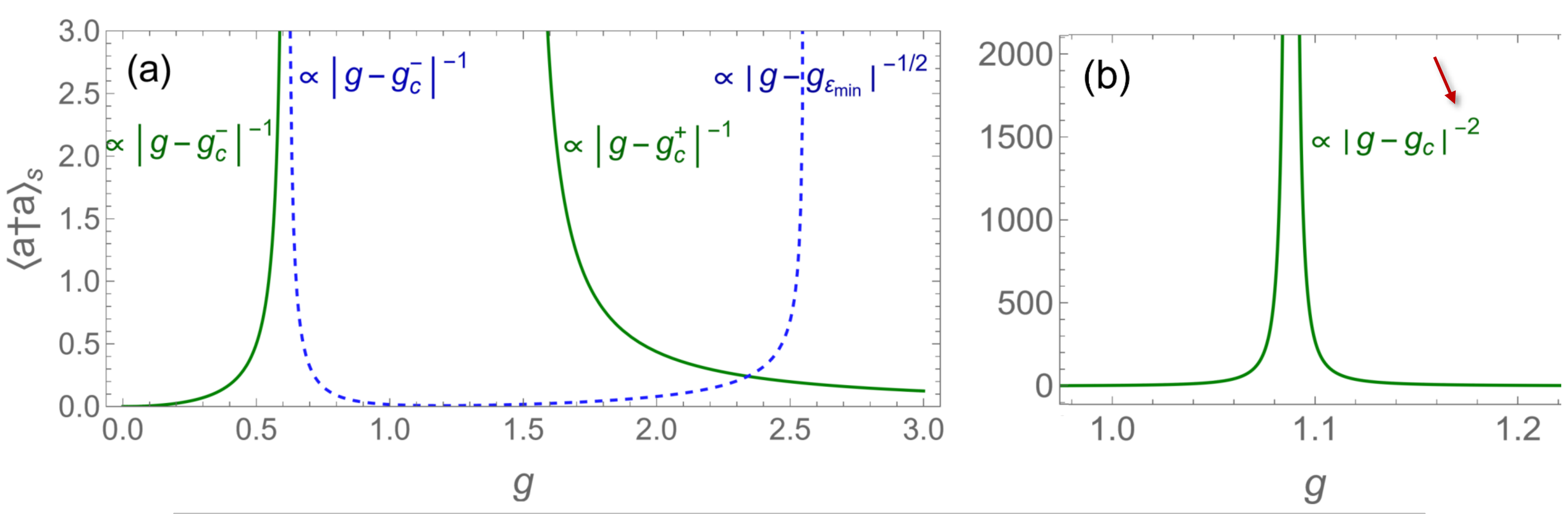


Asymptotic decay rate of the NP (solid line) and the SP (dashed line). (a) [(b)] is for the case g going along the blue [red] dotted line in Fig. 1(c).

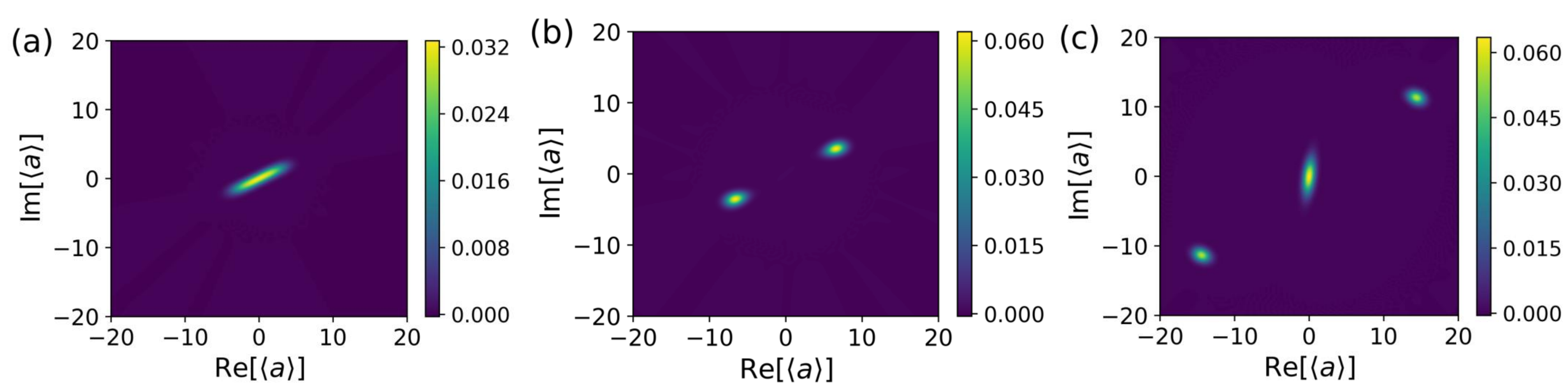
- Second moment of the oscillator operators:

$$\dot{\mathbf s} = M_{\rm np}\mathbf s + Y_{\rm np} \qquad \mathbf s = (\langle a^\dagger a\rangle, \langle a^2\rangle, \langle a^{\dagger 2}\rangle)^\intercal$$

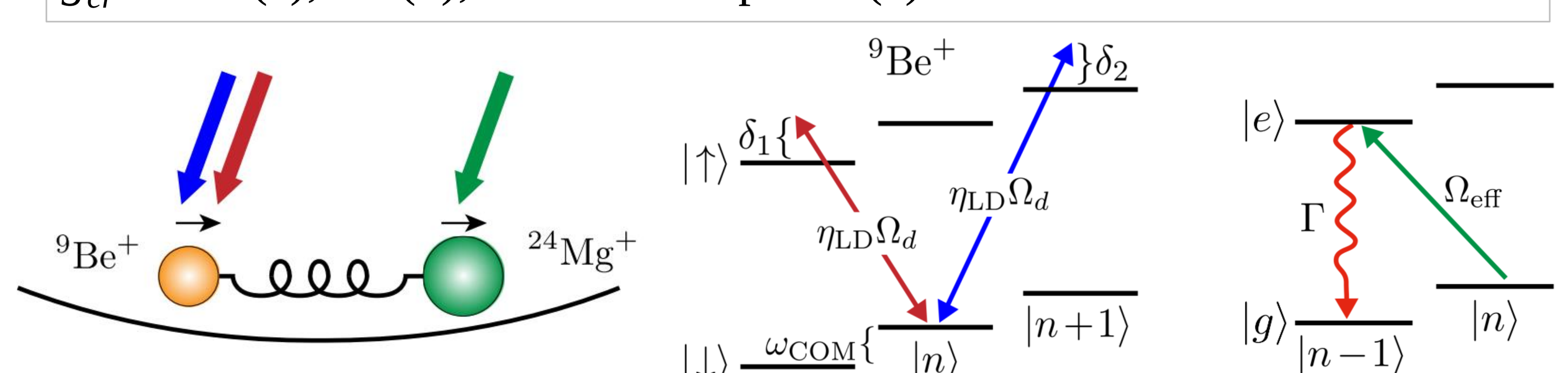
$$\langle a^\dagger a\rangle_s = \frac{2g^4\varepsilon^2}{g^4(1-\varepsilon^2)^2 - 2g^2(1+\varepsilon^2) + \bar\kappa^2 + 1}$$



The corresponding Scaling of the oscillator excitation number.



Numerical simulation of the master equation for finite Ω/ω_0. Wigner functions of the cavity field of the steady state for various g_r and g_{cr} in NP (a), SP (b), and bistable phase (c).



PRA2018 by Hwang *et.al.*

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