Dynamically-decoupled hyper-Ramsey spectroscopy of clock transitions

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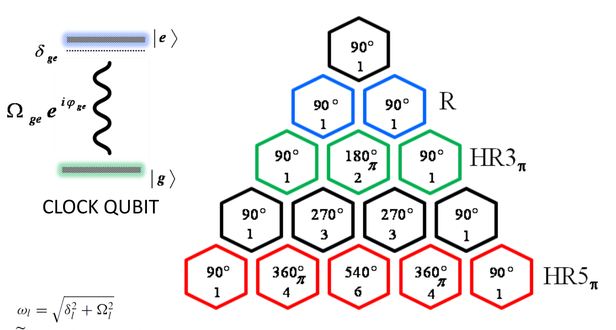
Introduction

- We present hyper-clocks that are designed to simultaneously eliminate probe-induced frequency-shifts and frequency-drifts for robust quantum sensing and frequency metrology, opening fault-tolerant laser spectroscopy for fundamental physics tests.

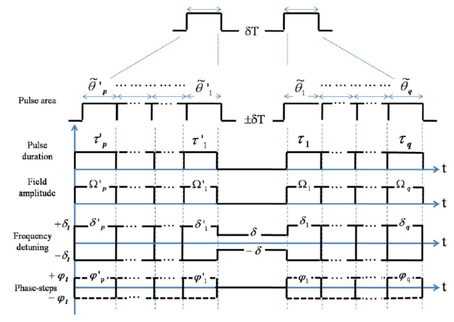
- Hyper-Ramsey spectroscopy is modified by inserting, halfway in between Ramsey pulses, a phase-shifted refocusing Hahn-echo pulse alternating frequency detunings with opposite signs during inter-pulse delays.
- Ultra-robust frequency error correction protocols can be achieved by combining dispersive interferometric signals eliminating residual frequency-shifts and frequency-drifts connected to a laser decoherence rate and slow intensity probe fluctuations.

→ Quantum interferences are shielded against light-shifts associated to a drastic reduction of a detrimental probe-laser frequency-drift.

I. SU(2) HYPER-CLOCKS: composite pulse spectroscopy



$$\omega_l = \sqrt{\delta_l^2 + \Omega_l^2}$$
$$\tilde{\theta}_l = \theta_l/2 = \omega_l \tau_l/2$$
$$\tilde{\vartheta}_l = \arcsin\left[\frac{\Omega_l}{\omega_l}\sin\tilde{\theta}_l\right]$$
$$\phi_l = \arctan\left[\frac{\delta_l}{\omega_l}\tan\tilde{\theta}_l\right]$$



$$U = {}^q_pC^*_{gg}\,{}^q_pC_{ge} + {}^q_pC_{gg}\,{}^q_pC^*_{ge},$$
$$V = i({}^q_pC^*_{gg}\,{}^q_pC_{ge} - {}^q_pC_{gg}\,{}^q_pC^*_{ge}),$$
$$W = |{}^q_pC_{gg}|^2 - |{}^q_pC_{ge}|^2.$$

$${}^q_pC_{gg} = {}^q_p\alpha_{gg}e^{i\delta T/2}\left[1 - |{}^q_p\beta_{gg}|e^{-i(\delta T + {}^q_p\Phi_{gg})}\right]$$
$${}^q_pC_{ge} = {}^q_p\alpha_{ge}e^{i\delta T/2}\left[1 + |{}^q_p\beta_{ge}|e^{-i(\delta T + {}^q_p\Phi_{ge})}\right]$$

$${}^q_p\alpha_{gg} = \alpha'^p_l(gg)\cdot\alpha^q_l(gg),$$
$${}^q_p\beta_{gg} = \beta'^p_l(gg)\cdot\beta^q_l(gg),$$
$${}^q_p\alpha_{ge} = \alpha'^p_l(ge)\cdot\alpha^q_l(gg),$$
$${}^q_p\beta_{ge} = \beta'^p_l(ge)\cdot\beta^q_l(gg).$$

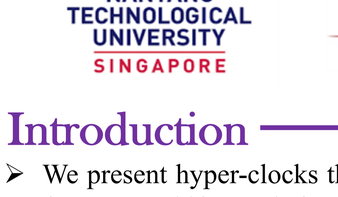
$${}^q_p\Phi_{gg} = \varphi_L + \phi_L - \mathrm{Arg}[{}^q_p\beta_{gg}]$$
$${}^q_p\Phi_{ge} = \varphi_L + \phi_L - \mathrm{Arg}[{}^q_p\beta_{ge}]$$

II. HYPER-RAMSEY INTERFERENCES:

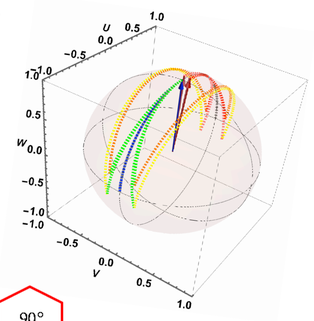
HR error-correction codes

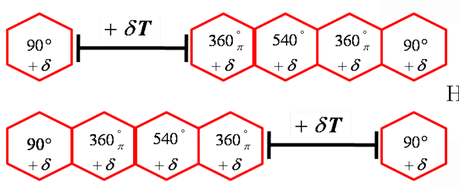
$${}^q_p\beta_{gg} = \beta'^p_l(gg)\cdot\beta^q_l(gg)$$

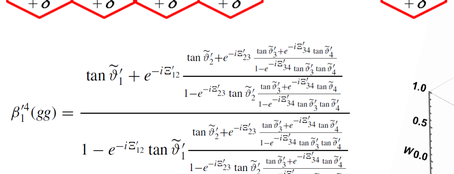
$$\beta'^1_1(gg) = \tan\tilde{\vartheta}'_1$$
$$\beta^1_1(gg) = \tan\tilde{\vartheta}_1$$



$$\beta'^2_1(gg) = \frac{\tan\tilde{\vartheta}'_1 + e^{-i\Xi'_{12}}\tan\tilde{\vartheta}'_2}{1 - e^{-i\Xi'_{12}}\tan\tilde{\vartheta}'_1\tan\tilde{\vartheta}'_2}$$
$$\beta^2_1(gg) = \frac{\tan\tilde{\vartheta}_1 + e^{-i\Xi_{12}}\tan\tilde{\vartheta}_2}{1 - e^{-i\Xi_{12}}\tan\tilde{\vartheta}_1\tan\tilde{\vartheta}_2}$$

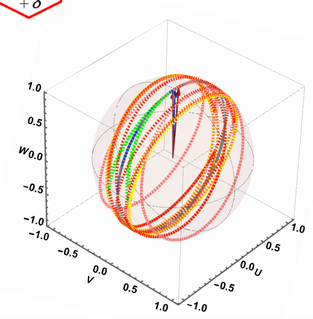


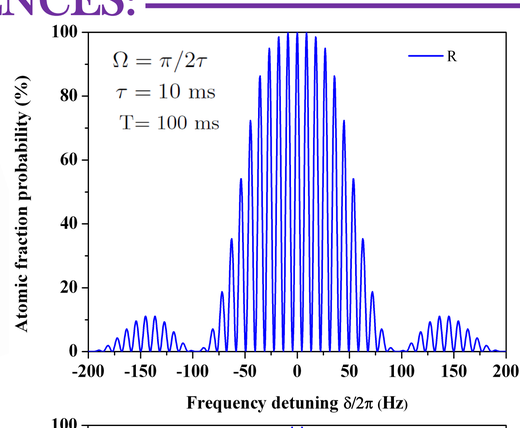


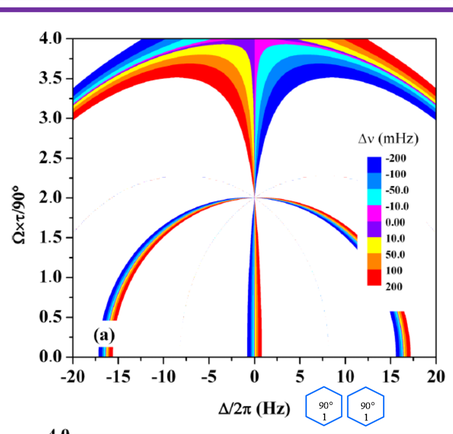


$$\beta'^4_1(gg) = \frac{\tan\tilde{\vartheta}'_1 + e^{-i\Xi'_{12}}\frac{\tan\tilde{\vartheta}'_2 + e^{-i\Xi'_{23}}\frac{\tan\tilde{\vartheta}'_3 + e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_4}{1-e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_3\tan\tilde{\vartheta}'_4}}{1-e^{-i\Xi'_{23}}\tan\tilde{\vartheta}'_2\frac{\tan\tilde{\vartheta}'_3 + e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_4}{1-e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_3\tan\tilde{\vartheta}'_4}}}{1 - e^{-i\Xi'_{12}}\tan\tilde{\vartheta}'_1\frac{\tan\tilde{\vartheta}'_2 + e^{-i\Xi'_{23}}\frac{\tan\tilde{\vartheta}'_3 + e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_4}{1-e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_3\tan\tilde{\vartheta}'_4}}{1-e^{-i\Xi'_{23}}\tan\tilde{\vartheta}'_2\frac{\tan\tilde{\vartheta}'_3 + e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_4}{1-e^{-i\Xi'_{34}}\tan\tilde{\vartheta}'_3\tan\tilde{\vartheta}'_4}}}}$$

$$\beta^4_1(gg) = \frac{\tan\tilde{\vartheta}_1 + e^{-i\Xi_{12}}\frac{\tan\tilde{\vartheta}_2 + e^{-i\Xi_{23}}\frac{\tan\tilde{\vartheta}_3 + e^{-i\Xi_{34}}\tan\tilde{\vartheta}_4}{1-e^{-i\Xi_{34}}\tan\tilde{\vartheta}_3\tan\tilde{\vartheta}_4}}{1-e^{-i\Xi_{23}}\tan\tilde{\vartheta}_2\frac{\tan\tilde{\vartheta}_3 + e^{-i\Xi_{34}}\tan\tilde{\vartheta}_4}{1-e^{-i\Xi_{34}}\tan\tilde{\vartheta}_3\tan\tilde{\vartheta}_4}}}{1 - e^{-i\Xi_{12}}\tan\tilde{\vartheta}_1\frac{\tan\tilde{\vartheta}_2 + e^{-i\Xi_{23}}\frac{\tan\tilde{\vartheta}_3 + e^{-i\Xi_{34}}\tan\tilde{\vartheta}_4}{1-e^{-i\Xi_{34}}\tan\tilde{\vartheta}_3\tan\tilde{\vartheta}_4}}{1-e^{-i\Xi_{23}}\tan\tilde{\vartheta}_2\frac{\tan\tilde{\vartheta}_3 + e^{-i\Xi_{34}}\tan\tilde{\vartheta}_4}{1-e^{-i\Xi_{34}}\tan\tilde{\vartheta}_3\tan\tilde{\vartheta}_4}}}}$$







$$\beta'^p_l(gg) = \frac{\tan\tilde{\vartheta}'_1 + e^{-i\Xi'_{12}}\frac{\tan\tilde{\vartheta}'_2 + e^{-i\Xi'_{23}}\frac{\tan\tilde{\vartheta}'_3 + \ldots}{1-\ldots}}{1-e^{-i\Xi'_{23}}\tan\tilde{\vartheta}'_2\frac{\tan\tilde{\vartheta}'_3 + \ldots}{1-\ldots}}}{1 - e^{-i\Xi'_{12}}\tan\tilde{\vartheta}'_1\frac{\tan\tilde{\vartheta}'_2 + e^{-i\Xi'_{23}}\frac{\tan\tilde{\vartheta}'_3 + \ldots}{1-\ldots}}{1-e^{-i\Xi'_{23}}\tan\tilde{\vartheta}'_2\frac{\tan\tilde{\vartheta}'_3 + \ldots}{1-\ldots}}}$$

$$\beta^q_l(gg) = \frac{\tan\tilde{\vartheta}_1 + e^{-i\Xi_{12}}\frac{\tan\tilde{\vartheta}_2 + e^{-i\Xi_{23}}\frac{\tan\tilde{\vartheta}_3 + \ldots}{1-\ldots}}{1-e^{-i\Xi_{23}}\tan\tilde{\vartheta}_2\frac{\tan\tilde{\vartheta}_3 + \ldots}{1-\ldots}}}{1 - e^{-i\Xi_{12}}\tan\tilde{\vartheta}_1\frac{\tan\tilde{\vartheta}_2 + e^{-i\Xi_{23}}\frac{\tan\tilde{\vartheta}_3 + \ldots}{1-\ldots}}{1-e^{-i\Xi_{23}}\tan\tilde{\vartheta}_2\frac{\tan\tilde{\vartheta}_3 + \ldots}{1-\ldots}}}$$

$${}^q_p\Phi_{gg} = \varphi_L + \phi_L - \mathrm{Arg}[{}^q_p\beta_{gg}]$$

ATOMIC PHASE-SHIFT EXTRACTION

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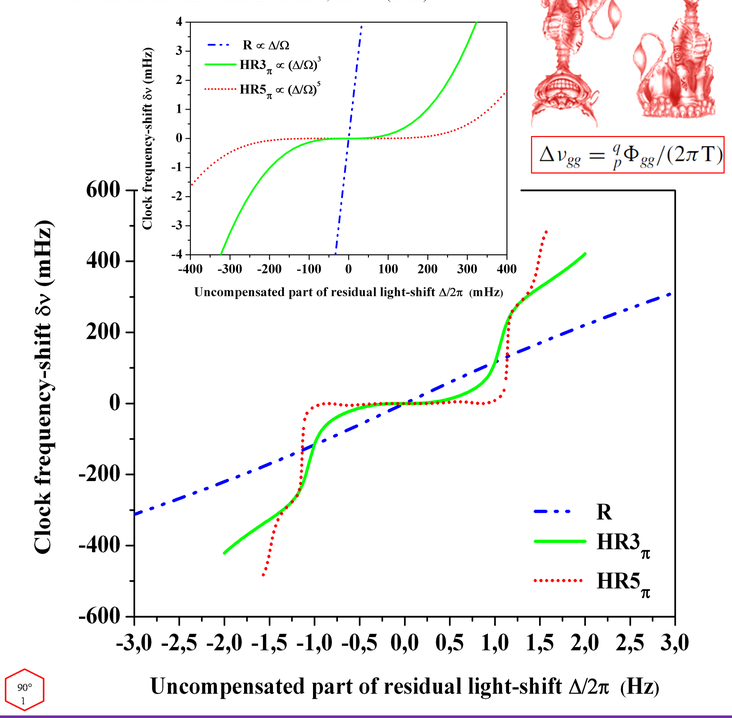
PHYSICAL REVIEW RESEARCH **4**, 023117 (2022)

N.F. Ramsey

$$\Xi_{1,l+1} = \varphi_{l+1} - \varphi_l + \phi_l + \phi_{l+1}$$
$$\Xi'_{l,l+1} = \varphi'_l - \varphi'_{l+1} + \phi'_l + \phi'_{l+1}$$

$$\varphi_L \equiv \varphi_1 - \varphi'_1$$
$$\phi_L \equiv \phi'_1 + \phi_1$$

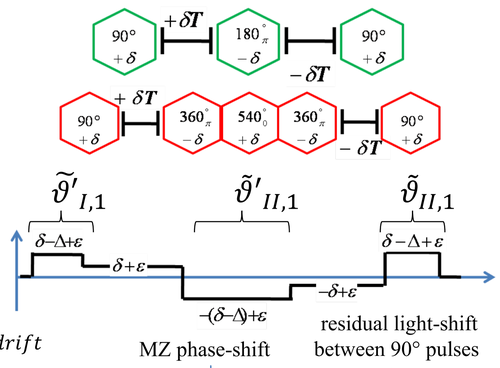
$$\Delta\nu_{gg} = {}^q_p\Phi_{gg}/(2\pi T)$$



II. DYNAMICALLY-DECOUPLED INTERFERENCES:

DD-HR algorithm

E.L. Hahn (spin echo)



$$\tilde{\vartheta}'_{I,1} \quad \tilde{\vartheta}'_{II,1} \quad \tilde{\vartheta}_{II,1}$$

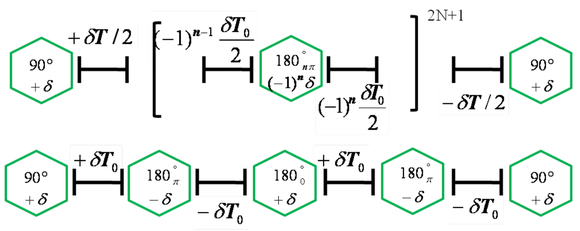
$$\varepsilon \equiv frequency - drift$$
$$\Delta \equiv light - shift$$

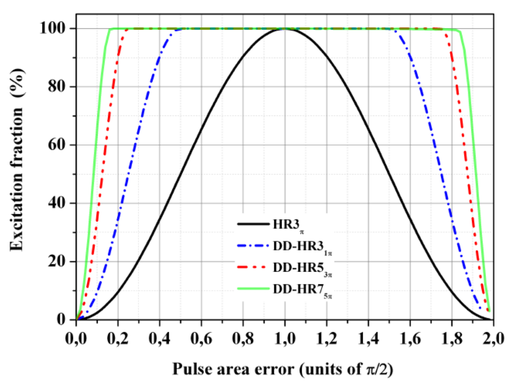
$$\Phi_{gg} = -\varphi'_{\mathrm{I},1} + 2\varphi'_{\mathrm{II},1} - \varphi_{\mathrm{II},1} + \phi'_{\mathrm{I},1} - \phi_{\mathrm{II},1}$$

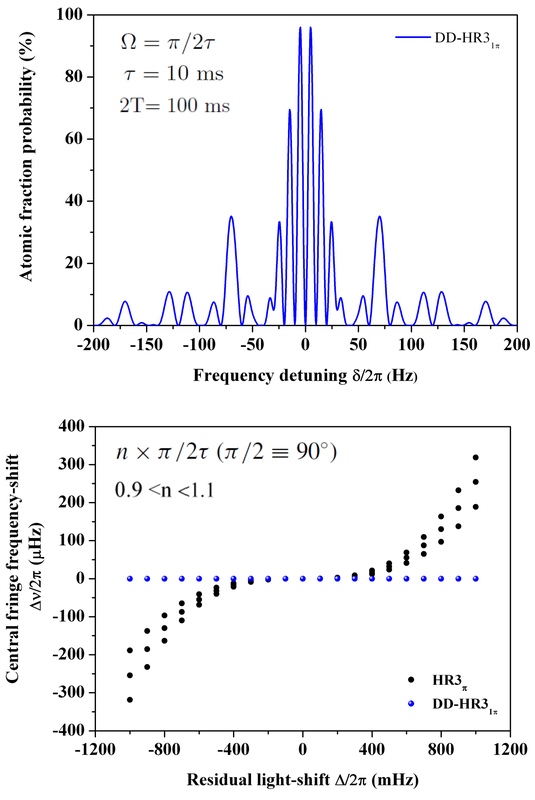
symmetric phase-alternating composite pulse algorithm

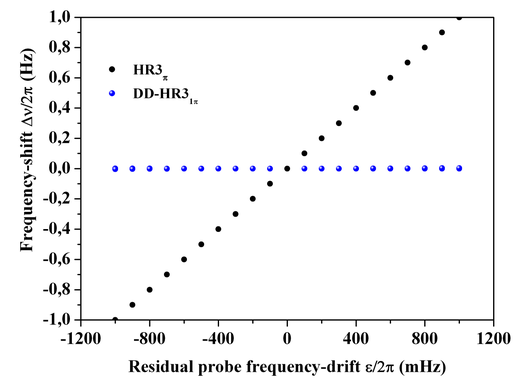
N∈{0,1,2,....} T≤ T_0

$$1 \le n \le 2N+1$$





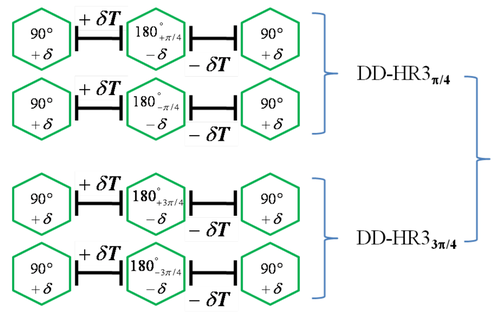


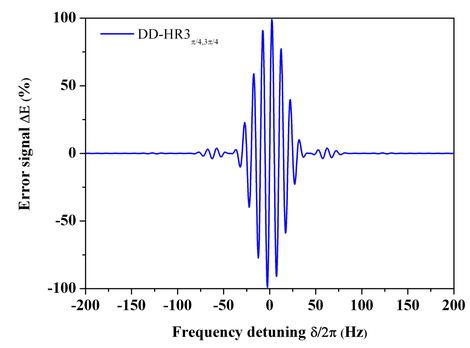


III. DD-HR SEQUENCING:

DD-HRS error-correction codes

$$\Delta E = P_g(\varphi^+_l) - P_g(\varphi^-_l)$$

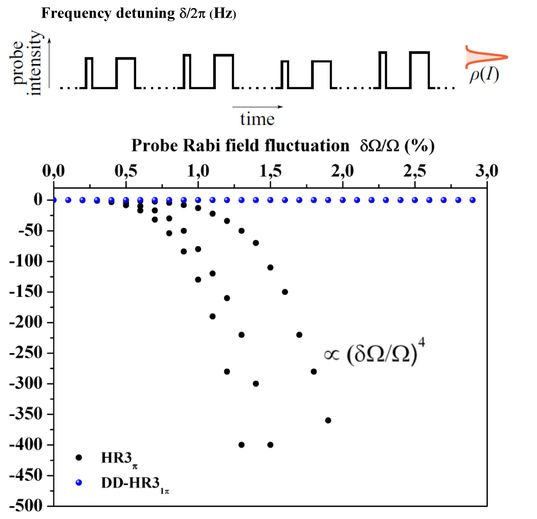


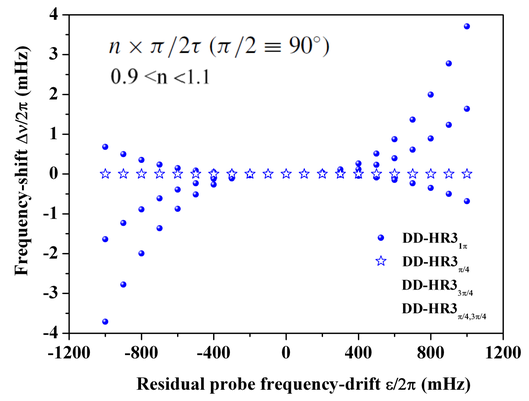


F. Bloch (OBEs)

Rep. Prog. Phys. **81** (2018) 094401
PHYSICAL REVIEW A **96**, 023408 (2017)

$$\dot{U}_l = -\gamma_c U_l + \delta_l V_l - \Omega_l \sin\varphi_l W_l,$$
$$\dot{V}_l = -\delta_l U_l - \gamma_c V_l + \Omega_l \cos\varphi_l W_l,$$
$$\dot{W}_l = \Omega_l \sin\varphi_l U_l - \Omega_l \cos\varphi_l V_l$$





Conclusions

- Dynamically-decoupled hyper-Ramsey spectroscopy with phase-alternating composite pulses can be designed to operate within various experimental quantum simulation platforms suffering from noisy environment and technical distortions synchronized with electromagnetic pulses.

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- K. Beloy, *Hyper-Ramsey spectroscopy with probe-laser intensity fluctuations*, Phys.Rev. A **97**, 031406(R) (2018).