Probing topological phase transition and critical physics in a synthetic quantum Hall system

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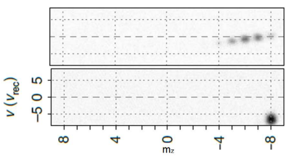
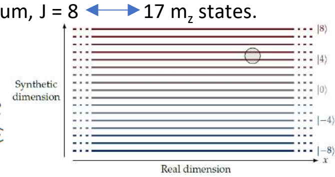
Why Dysprosium?

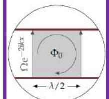
^{162}Dy Isotope (**Boson**) in its GS: [Xe]4f^{10}6s^2(^{5}I$_8$) has L = 6, S = 2 and I = 0. Total angular momentum, J = 8 ⟷ 17 m_z states.



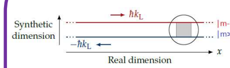
Absorption imaging:
TOF expansion (+ magnetic gradient) = Single site resolution in synthetic dimension



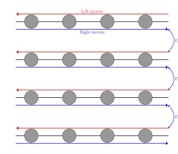
Our system:
~ 3.10^4 ^{162}Dy atoms at 0.2 μK initially polarized in |-8>$_z$



2D Synthetic Quantum Hall (QH) System



Compare with the coupled wire picture of Integer QH



To couple the wires (m_z states) we use two photon **Raman transitions** via off-resonant 626 nm light

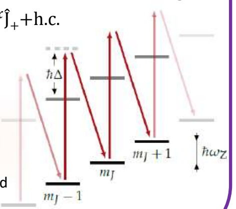
$$\hat{H}_{Dy} = \frac{1}{2}Mv^2 - te^{-i2\hbar k_L x}\hat{J}_+ + \text{h.c.}$$
$$= \frac{(\hat{p}_x - 2\hbar k_L \hat{J}_z)^2}{2M} - \hbar U \hat{J}_x$$

⟷

$$\hat{H}_{Landau} = \frac{(\hat{p}_x - eB\hat{y})^2}{2M} - \frac{\hat{p}_y^2}{2M}$$

With recoil momentum $\hbar k_L$ and Raman coupling $\hbar U$

$\hat{J}_z \sim \hat{y}$, $2\hbar k_L \sim eB$, $\hat{J}_x \sim \hat{p}_y^2$

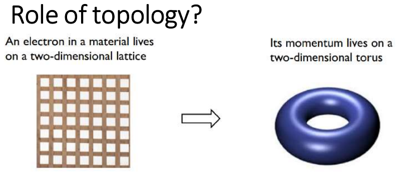


B

ω — u_1 — Dy Atom — $\omega + \omega_z + \delta$ — u_2

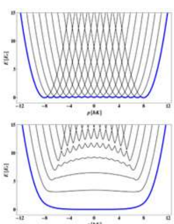
Topological Probes

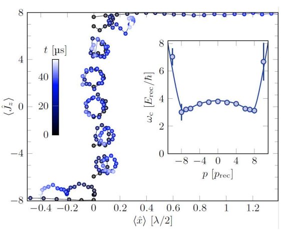
Role of topology?

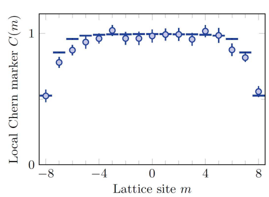


Its probes in QH systems?

- Lowest Landau Level
- Ballistic chiral edge modes
- Cyclotron and skipping orbits
- Hall conductivity (local response)
- Local Chern Marker

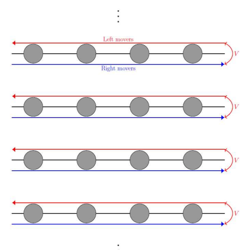






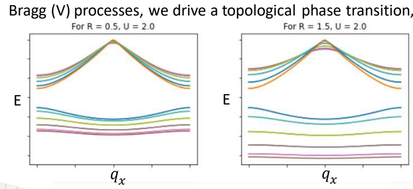
Chalopin et al. Probing chiral edge dynamics and bulk topology of a synthetic Hall system. Nat. Phys 2020

Cosine lattice and Topological Phase Transition

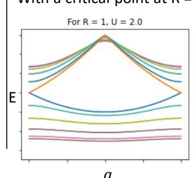


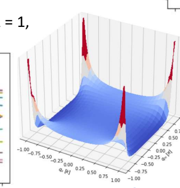
$$\hat{H}_{Dy} = \frac{1}{2}Mv^2 - te^{-i2\hbar k_L x} + \text{h.c.}$$
$$= \frac{(\hat{p}_x - 2\hbar k_L \hat{J}_z)^2}{2M} - \hbar V \text{Cos}(2\hbar k_L x)$$

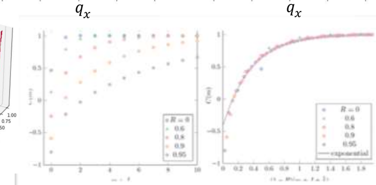
By varying the ratio (R = U/V) of power between Raman (U) and Bragg (V) processes, we drive a topological phase transition,



With a critical point at R = 1,

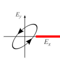






B

ω — Dy Atom — $\omega + \omega_z + \delta$ — $\omega + \delta$



Robustness of C = 1/2

Berry curvature close to the critical point,

$$F(q_x, q_m) = F_{reg}(q_x, q_m) + \delta F(q_x, q_m)$$

has a diverging contribution δF and a regular contribution that corresponds to C = 1/2

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} dq_m \int dq_x\, F_{reg}(q_x, q_m) = \frac{1}{2}$$

The diverging component corresponds to correlation lengths that diverge as,

$$\xi_x = \frac{4}{Uk}\frac{1}{1-R}, \quad \xi_m = \frac{1}{1-R}$$

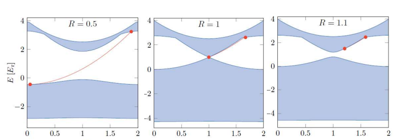
This half-quantization appears because of emergent parity symmetry at the Dirac point, $R = 1$ and $\hat{q}_m = \pi$,

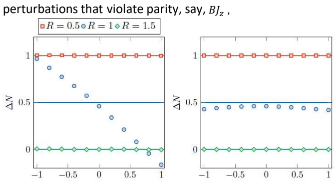
$$\hat{H}_{Dy} = \frac{(\hat{q}_x - 2\hbar k_L \hat{J}_z)^2}{2M} - \hbar V \text{Cos}(2\hbar k_L x) + \hbar U(e^{-i2\hbar k_L x}\hat{J}_+ + e^{i2\hbar k_L x}\hat{J}_-)$$
$$\approx \frac{\hat{q}_x^{\,2}}{2M} + \frac{\hbar^2 k_L(\hat{q}_x - \hbar k_L)}{M}\sigma_z - \hbar U(\hat{q}_m - \pi)\sigma_y + \hbar U(1-R)\sigma_x$$

Away from the Dirac point, this symmetry corresponds to,

$$x \to x + \frac{\lambda}{4}$$
$$m \to -m$$

Therefore, we expect robustness for perturbations that preserve this symmetry, for example, a quadratic Zeeman shift, QJ_z^2 and non-quantization for perturbations that violate parity, say, BJ_z.



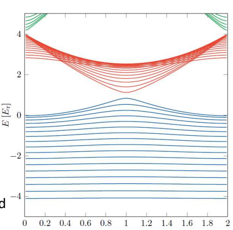


State preparation

Non-uniform Clebsch-Gordon coefficients with $\hat{J}_+$ necessitate,

$$\hat{H}_{Dy} = \frac{(\hat{q}_x - 2\hbar k_L \hat{J}_z)^2}{2M} - \hbar V\left(1 - \frac{J_z^2}{2J^2}\right)\text{Cos}(2\hbar k_L x) + \frac{\hbar U}{2J}(e^{-i2\hbar k_L x}\hat{J}_+ + e^{i2\hbar k_L x}\hat{J}_-)$$

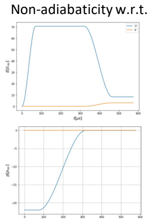
Experimentally, this corresponds to elliptical polarization for Raman and Bragg beams, with $\theta = \cos^{-1} 0.84$ and $\varphi = \pi/2$, $\boldsymbol{\varepsilon} = \begin{pmatrix} e^{i\varphi}\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$



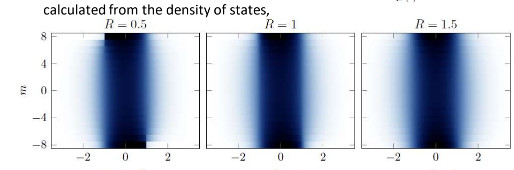
State preparation requires,

- Adiabaticity w.r.t. bands
- Non-adiabaticity w.r.t. sub-bands

→ "Semi-metallic" behavior with an incoherent mixture of 17 sub-bands in the ground band manifold



Various correlation functions can be calculated from the density of states,



Summary

- **Topological phase transition**: Theoretically challenging because of diverging correlation lengths. This is specially true for topological phase transitions where there is no clear order parameter (see our previous work: arXiv:2307.06251). Towards this end, this is the first study of topological phase transition using ultracold atoms.
- **Half-quantized topological response:** Chern number (or Chern marker in this case) is a TKNN invariant and only takes integer values. However, at the critical point, it is possible to have non-integer values. In this work, we experimentally demonstrate the half-integer quantized topological response. Furthermore, we demonstrate the robustness of this half-quantized response. This is also potentially interesting from quantum computing perspective.
- **Bulk Conformal Field Theory:** We were able to characterize certain correlation functions of the bulk critical theory by looking at the correlation lengths and the density of states. While this characterization is not enough to give us complete information about the critical point, finiteness of the c < 1 rational CFT models restricts the number of possibilities.

Outlook

- **Addition of disorder:** While the system studied in this experiment is analytically solvable, it would be interesting to study it in presence of disorders. In particular, the critical exponents of integer quantum Hall transition are not very well understood*. We could add disorder to our current experiment using standard techniques like adding incommensurate optical lattices.
- **Vortices:** Topological defects like quantized vortices are a characteristic property of superfluids. As such, it will interesting to observe quantized vortices that are produced upon quenching the lattice strength, V.
- **Bulk CFT and Edge chiral CFTs:** Topological systems exhibit Bulk-edge correspondence, wherein, the bulk topology is in one-to-one correspondence with the edge chiral CFT (or Luttinger liquid). We have studying some properties of the edge CFT in our last paper, arXiv:2307.06251. It would be interesting to understand how the edge CFTs change during the topological phase transition and what is their relationship with the bulk CFT that emerges at the critical point.

*https://journals.aps.org/prb/abstract/10.1103/PhysRevB.80.041304 and its citations