Influence of catastrophes and hidden dynamical symmetries on ultrafast backscattered photoelectrons

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Abstract

In this work, we discuss the effect of using a soft-core Coulomb potential in the photoelectron momentum distribution (PMD) using the hybrid forward-boundary CQSFA (H-CQSFA). We show that introducing a softening in the Coulomb interaction influences the ridges observed in the PMDs associated with backscattered electron trajectories. For a hard-core Coulomb interaction, the re-scattering ridges close along the polarization axis, while for a soft-core potential, they are interrupted at ridge-specific angles, which depend on the potential softening. We analyze the momentum mapping of the different orbits leading to the ridges.

Interaction of a strong laser pulse with a model hydrogen atom

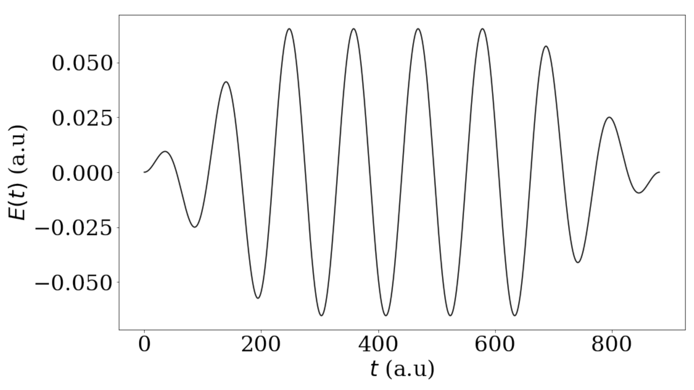
Hamiltonian

$H(t) = \frac{\hat{\mathbf{p}}^2}{2} + V(\hat{\mathbf{r}}) + \hat{\mathbf{r}} \cdot \mathbf{E}(t)$

Soft-core Coulomb potential.

$V(\mathbf{r}) = -1/\sqrt{\mathbf{r}^2 + \alpha^2}$

Laser field: $E(t) = f(t)E_0 \sin(\omega t)$

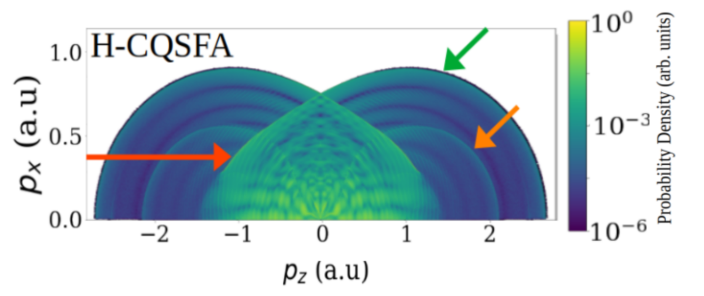


$\lambda = 800$ nm, $I = 1.5 \times 10^{14}$ W/cm^2

Caustics

- Multiple rays of light can coalesce and form bright focusing features which are known as caustics [1].
- We can see them at ridges and elsewhere in orbit-based methods.
- Present due to failures in the semi-classical approximation.

Photoelectron momentum distribution



CQSFA transition amplitude and saddle-point equations

The Coulomb Quantum-Orbit Strong Field approximation (CQSFA) is a method based on the propagation of Coulomb-distorted quantum orbits[2].

CQSFA transition amplitude

$$M(\mathbf{p}_f) \propto -i \lim_{t\to\infty} \sum_s \left\{ \det \left[\frac{\partial \mathbf{p}_s(t)}{\partial \mathbf{p}_s(t'_s)} \right] \right\}^{-1/2} \mathcal{C}(t'_s) e^{iS(\tilde{\mathbf{p}}_s, \mathbf{r}_s, t, t'_s)}$$

Saddle Point Equations

$$-2I_p = [\mathbf{p}(t') + \mathbf{A}(t')]^2$$
$$\dot{\mathbf{r}}(\tau) = \mathbf{p}(\tau) + \mathbf{A}(\tau)$$
$$\dot{\mathbf{p}}(\tau) = -\nabla_r V(\mathbf{r}(\tau))$$

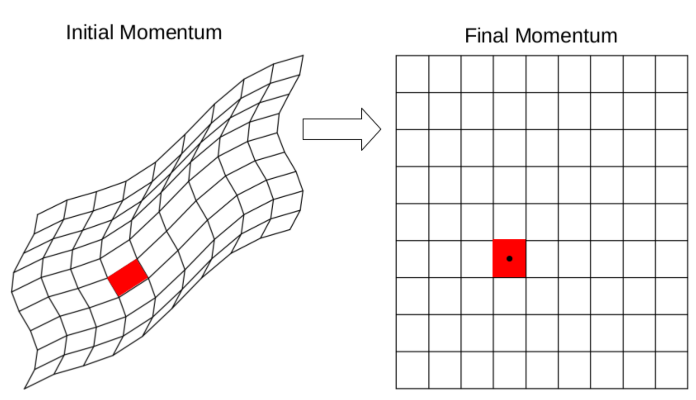
- Caustics are present in the probability distribution when the determinant approaches zero.
- Focal points are the points along a trajectory such that this term vanishes.

Orbit	Π_z	Π_x	Behavior
1	+	+	Direct
2	-	+	Hyperbola
3	-	-	Hyperbola
4	+	-	Rescattered

Hybrid Forward-Boundary CQSFA

To evaluate the transition amplitude we need to find a subset of the initial momenta, that are propagated to specified momentum values using the saddle point equations.[3]

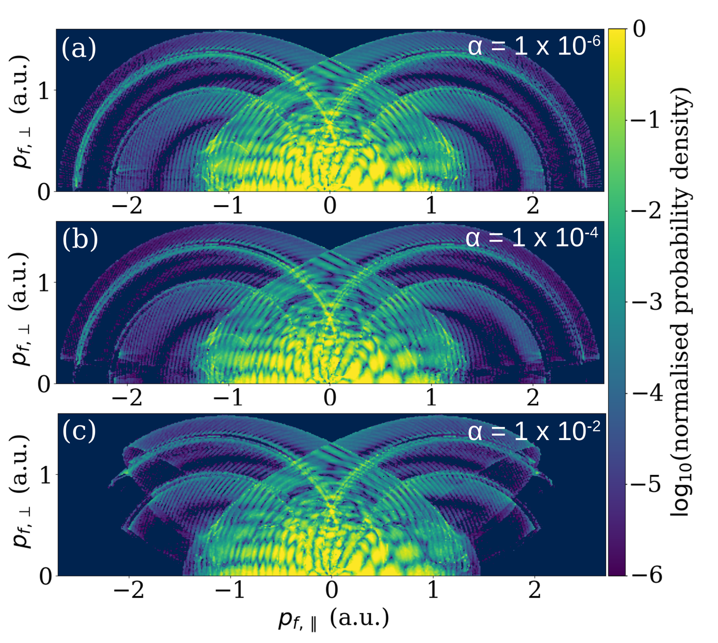
- Sample the initial momentum and obtain the initial positions and ionization times.
- Propagate the electrons solving the saddle-point equations and find the asymptotic momenta.
- Bin the final momentum, keeping only one trajectory within each bin, removing duplicates.



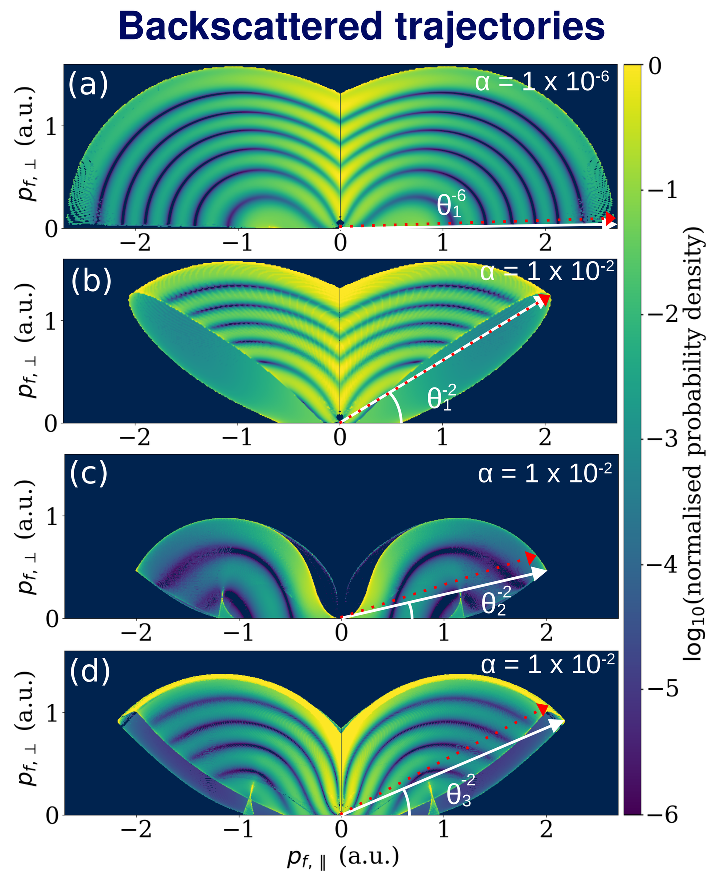
Photoelectron Momentum Distributions and backscattered trajectories

Variable softening parameter

- Hard-core Coulomb potential ($\alpha = 10^{-6}$)
- Soft-core Coulomb potential ($\alpha = 10^{-4}$, and $\alpha = 10^{-2}$)



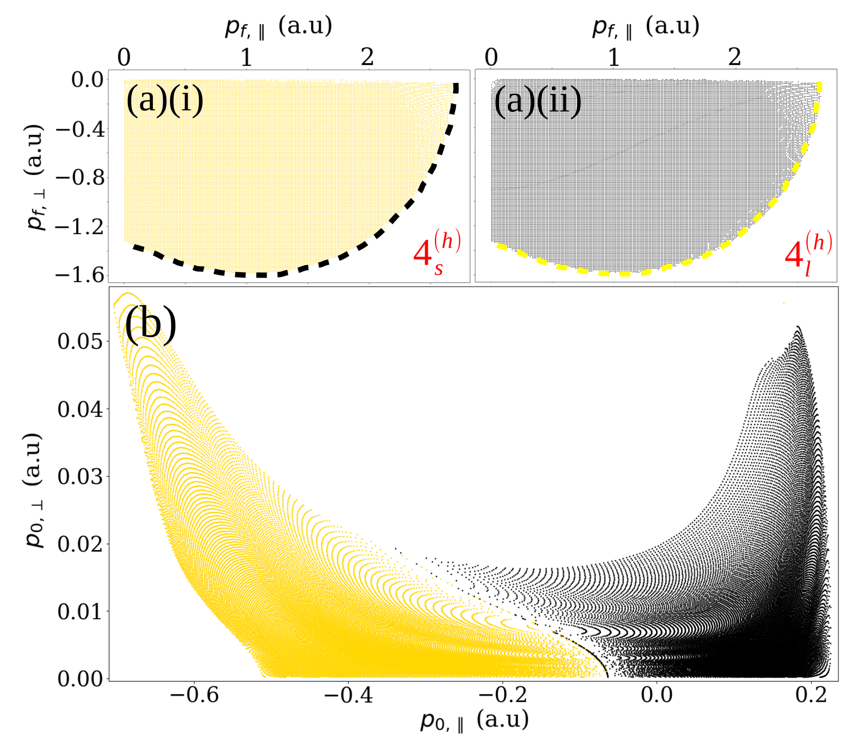
Backscattered trajectories



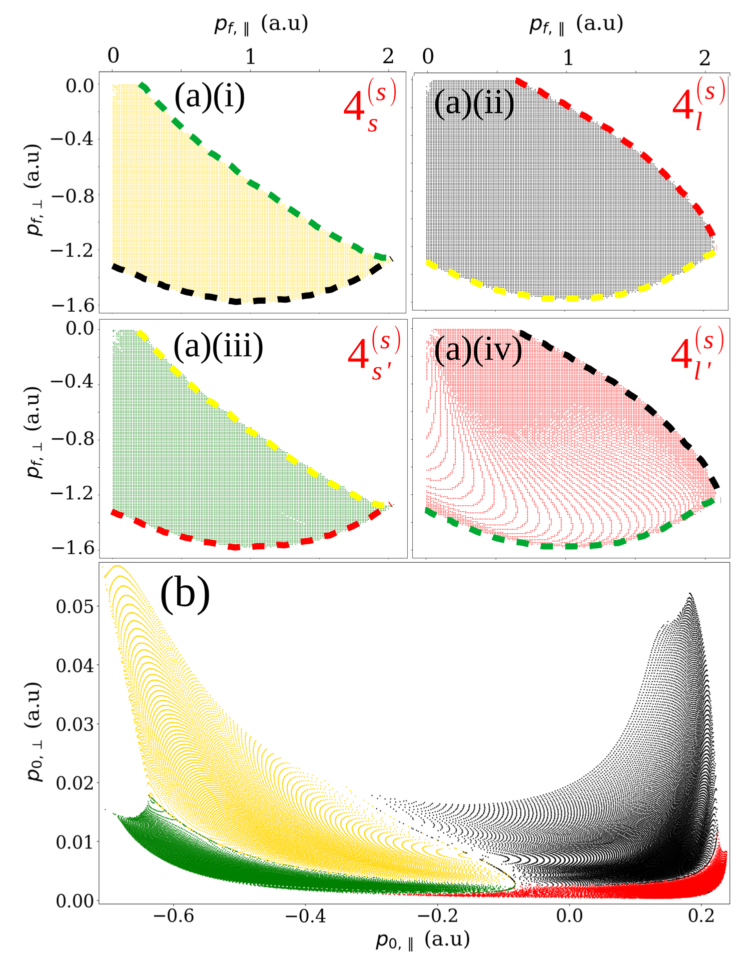
- The PMDs calculated using the hard-core potential, show rescattering ridges extending up to the laser polarization axis.
- As the softening increases, the ridges seem to fold around a minimal rescattering angle.
- Both the ridges' energies and the folding angles vary according to the specific orbit pair.

Momentum mapping

Hard-core $\alpha = 10^{-6}$



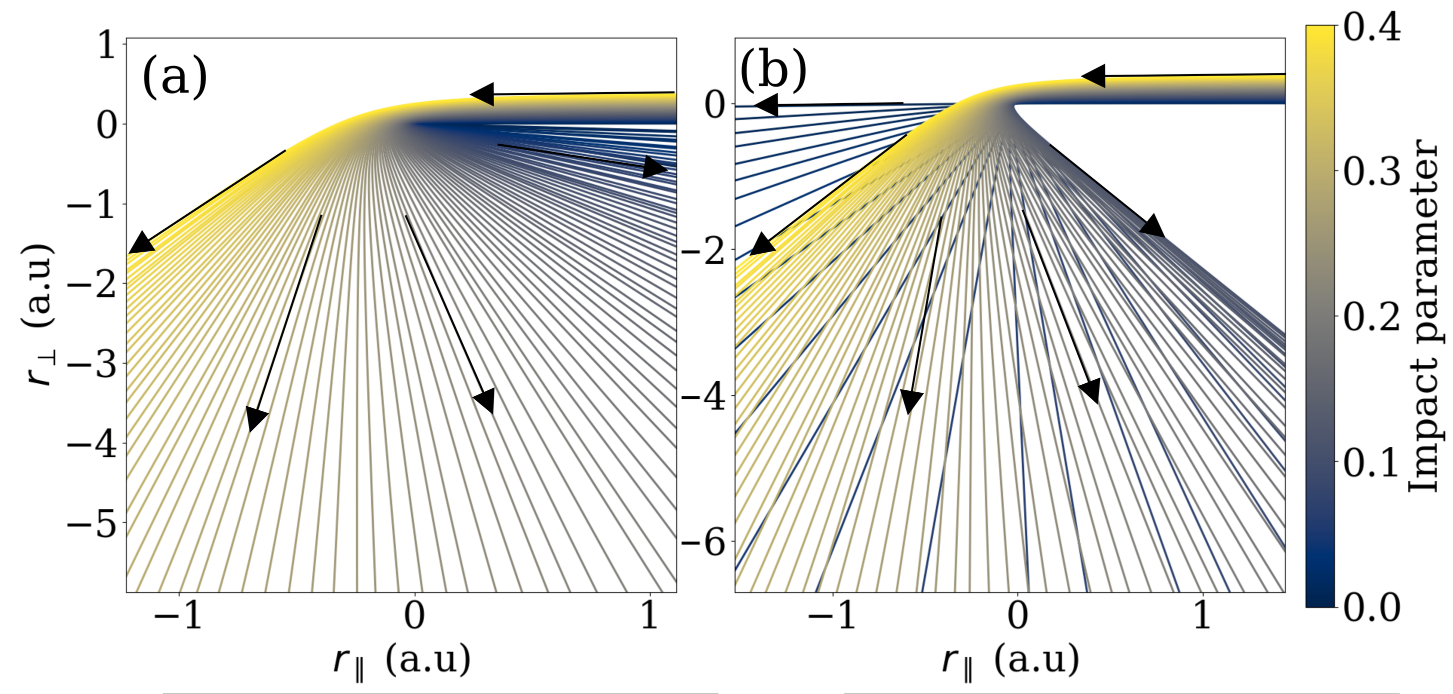
Soft-core $\alpha = 10^{-2}$

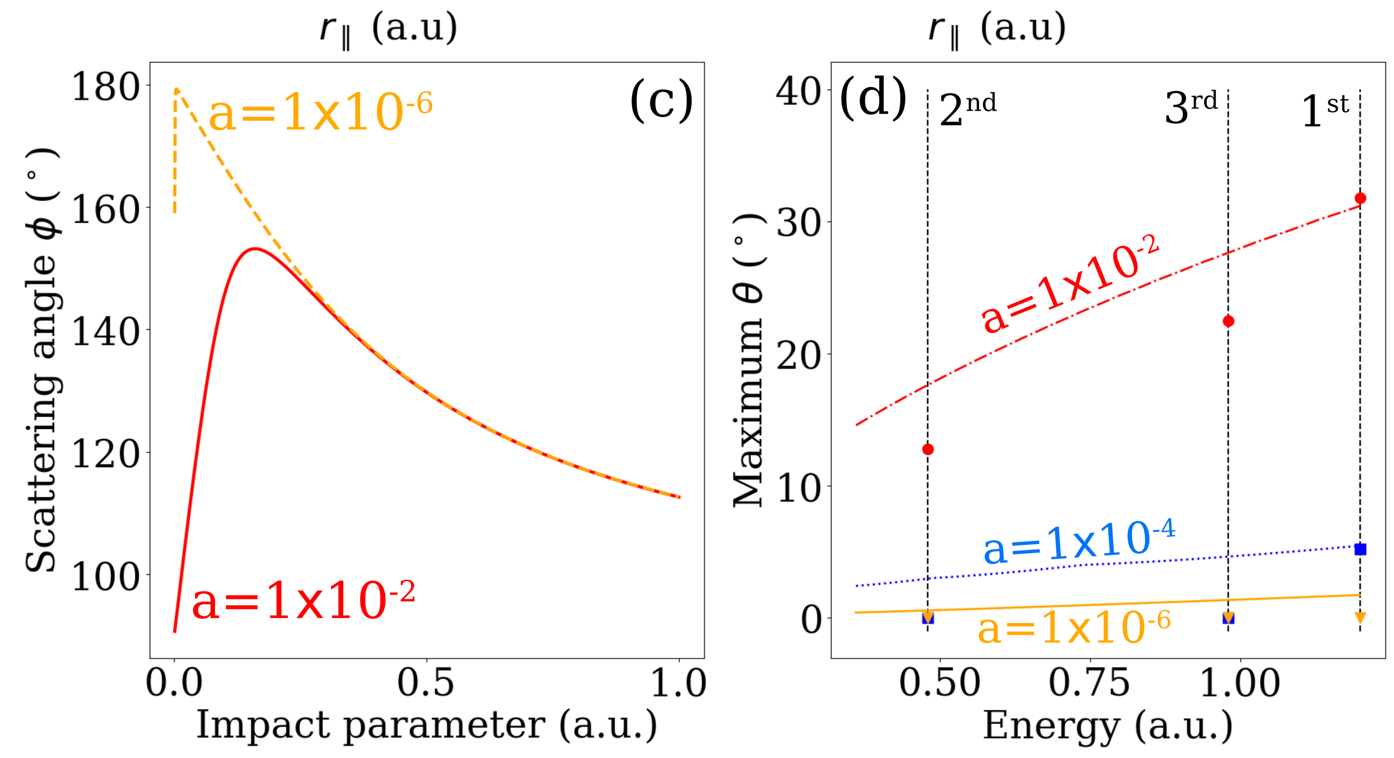


Upper panel: final momenta. **Lower panel:** initial momenta. The colours indicate the different solutions contributing to the ridges.

- The boundaries of the sheets are delineated by caustics.
- Two extra sheets of solutions appear as the softening parameter is increased.
- A new caustic appears, because a new fold has formed in the momentum mapping.

Scattering model





- For a hard-core potential, the electron can be scattered up to an angle close to π.
- For a soft-core potential there is a maximum scattering angle, and as the impact parameter is reduced further some trajectories pass through the core.

Concluding remarks and perspectives

- The rescattering ridges don't close at the polarization axis due to the softened potential, and we observe different angles, specific to each ridge and softening parameter.
- New solutions to the saddle point equations appear when the softening is increased as the ridge folds in on itself.
- We show how our results can be explained with a simple scattering model.

Bibliography

(1) O Raz, Nature Photonics, 6, 170 (2012).
(2) X.-Y. Lai et al, Phys. Rev. A **92**, 043407 (2015).
(3) L. Cruz Rodriguez, et al, Phys. Rev. A 108, 033114 (2023).