Collective excitations and nonequilibrium dynamical phase transition in dissipative fermionic superfluids

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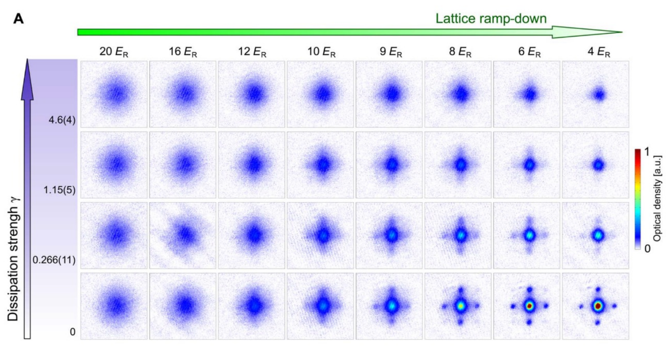
1. Introduction

Background 1 : Quantum systems with dissipation in experiments

E.g. **Superfluid-Mott** transition by **dissipation**

Tomita, *et. al.*, Science advances 2017



Interference pattern of superfluid

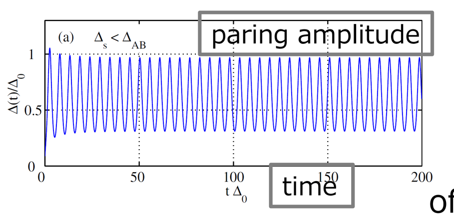
Dissipation

Unclear

Background 2 : Collective excitaions in condensed matter physics

Higgs mode PRL **96**, 230403 (2006) PRB **92**, 064508 (2005)

$\omega_H = 2\Delta$

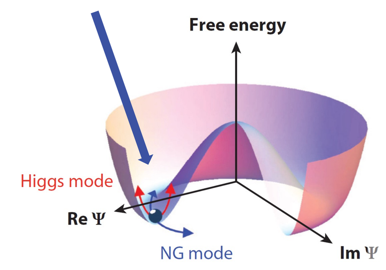


Sudden change of the interaction

Amplitude oscillation of superfluid order parameter

Nambu-Goldstone mode

Gapless phase mode required by symmetry



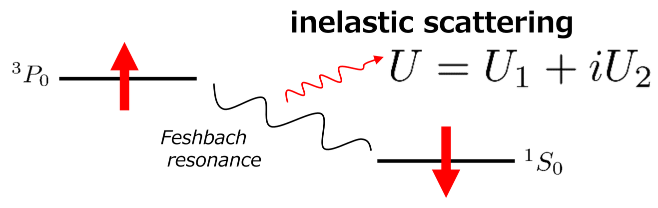
Annu. Rev. Condens. Matter Phys. **11**, 103 (2020)

Collective excitations in ultracold atoms

Suffer from **atom loss** due to **inelastic scattering**

Experiment: e.g. *Orbital Feshbach resonance*

→ Inelastic collision between different orbitals



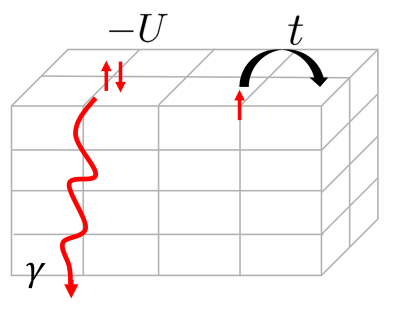
Our study: fermionic superfluidity + two-body loss

⇒ Order parameter with a ***complex-valued*** interaction

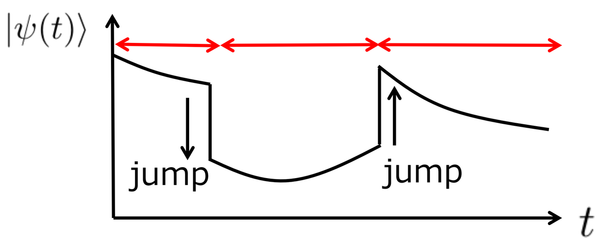
2. Model & Methods

Three-dimensional attractive Hubbard model + two-body loss

$$H = \sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - U_{\rm R} \sum_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow} c_{i\uparrow}\,, \quad L_i = c_{i\downarrow} c_{i\uparrow}$$



Lindblad master equation



Von Neuman equation — dissipation — Coupling to the environment

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i[H,\rho] - \frac{\gamma}{2}\sum_i (\{L_i^\dagger L_i, \rho\} - 2L_i \rho L_i^\dagger)$$

NH term — Jump term

Dissipative BCS theory

Generating functional

$$Z = {\rm tr}\rho = \int \mathcal{D}[c_-, \bar{c}_-, c_+, \bar{c}_+] e^{iS} = 1$$

$$S = \int_{-\infty}^{\infty} dt \left[\sum_{\boldsymbol{k}\sigma} (\bar{c}_{\boldsymbol{k}\sigma+} i\partial_t c_{\boldsymbol{k}\sigma+} - \bar{c}_{\boldsymbol{k}\sigma-} i\partial_t c_{\boldsymbol{k}\sigma-}) - H_+ + H_- + \frac{i\gamma}{2}\sum_i (\bar{L}_{i+}L_{i+} + \bar{L}_{i-}L_{i-} - 2L_{i+}\bar{L}_{i-}) \right]$$

Hubbard-Stratonovich transformation

ψ: Nambu spinor

$$S = \int dt \left[\sum_{\boldsymbol{k}} \left\{ \bar{\psi}^t_{\boldsymbol{k}+} \begin{pmatrix} i\partial_t - \epsilon_{\boldsymbol{k}} & -\Delta \\ -\Delta^* & -i\partial_t + \epsilon_{\boldsymbol{k}} \end{pmatrix} \psi_{\boldsymbol{k}+} - \bar{\psi}^t_{\boldsymbol{k}-} \begin{pmatrix} i\partial_t - \epsilon_{\boldsymbol{k}} & -\Delta \\ -\Delta^* & -i\partial_t + \epsilon_{\boldsymbol{k}} \end{pmatrix} \psi_{\boldsymbol{k}-} \right\} \right]$$

Order parameter (saddle point condition)

$$\Delta = -\frac{U}{N_0}\sum_{\boldsymbol{k}} {\rm tr}(c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow}\rho) \equiv -\frac{U}{N_0}\sum_{\boldsymbol{k}} \langle c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow}\rangle$$

$$U = U_R + \frac{i\gamma}{2}$$

Time evolution of the **density matrix**

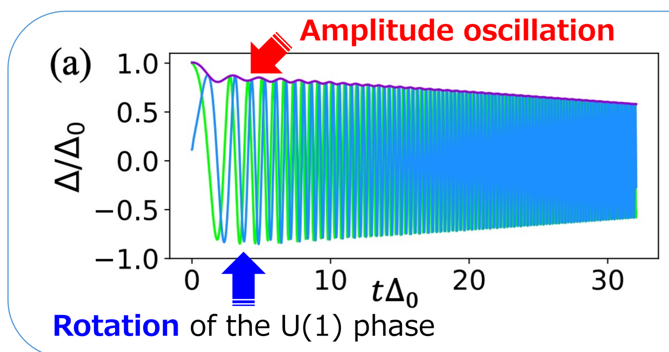
$$\frac{d\rho}{dt} = -i[H_{\rm eff}, \rho]$$

$$H_{\rm eff} = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \begin{pmatrix} \epsilon_{\boldsymbol{k}} & \Delta \\ \Delta^* & -\epsilon_{\boldsymbol{k}} \end{pmatrix} \Psi_{\boldsymbol{k}}$$

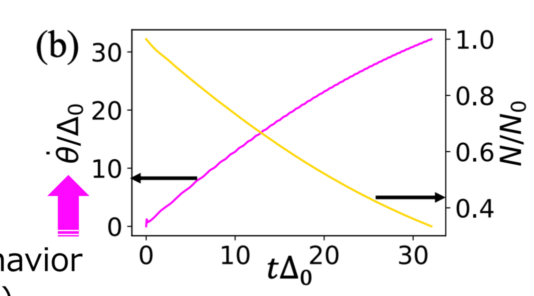
3. Results

Numerical solutions : amplitude and phase mode

■ **Dynamics of superfluid after dissipation is introduced (quench dynamics)**



Rotation of the U(1) phase



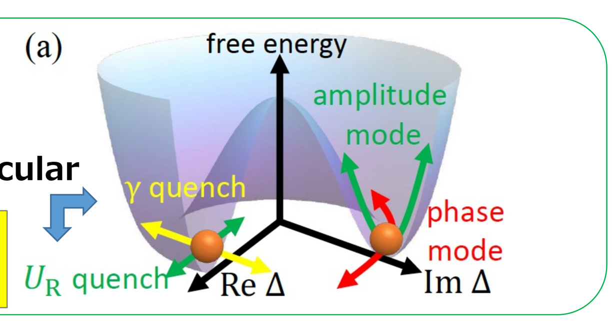
chirping behavior (increasing θ)

Reduction of the particle #

Phase rotation ?? Amplitude oscillation ??

perpendicular

$U = U_R + \frac{i\gamma}{2}$



chirping?? $\frac{d\theta}{dt} = -2\mu_{\rm eff}$

Reduction of the particle #

Reduction of the effective chemical potential

U(1) phase rotation highlights the dissipative superfluid

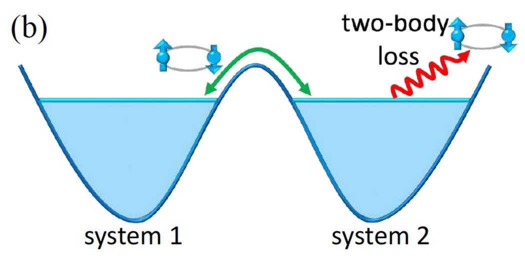
but… can be gauged out w/ time-dependent effective chemical potential

$$\Delta(t) = \exp(-2i\int_0^t \mu_{\rm eff}(t)dt)\Omega(t) \quad (\Omega(t) \in \mathbb{R})$$

Gauge-invariant physical observable ?? ⇐ **Experimentally accessible**

Two superfluids connected via **Josephson junction**

Relative phase is **gauge-invariant** !



$$H_{\rm sys} = H_1 + H_2 + H_{\rm tun}$$

$$H_i = \sum_{\boldsymbol{k}} \Psi^\dagger_{i\boldsymbol{k}} \begin{pmatrix} \epsilon_{\boldsymbol{k}} & \Delta_i \\ \Delta_i^* & -\epsilon_{\boldsymbol{k}} \end{pmatrix} \Psi_{i\boldsymbol{k}}$$

$$H_{\rm tun} = -\frac{V}{N_0}\sum_{\boldsymbol{k}\boldsymbol{k}'} (c^\dagger_{1\boldsymbol{k}\uparrow} c^\dagger_{1-\boldsymbol{k}\downarrow} c_{2-\boldsymbol{k}'\downarrow} c_{2\boldsymbol{k}'\uparrow} + {\rm H.c.})$$

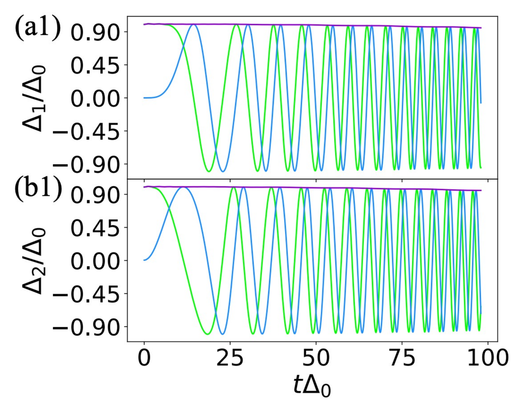
$$\Delta_1 = -\frac{U_{\rm R}}{N_0}\sum_{\boldsymbol{k}} \langle c_{1-\boldsymbol{k}\downarrow} c_{1\boldsymbol{k}\uparrow}\rangle$$

$$\Delta_2 = -\frac{U}{N_0}\sum_{\boldsymbol{k}} (\sigma^x_{2\boldsymbol{k}} - i\sigma^y_{2\boldsymbol{k}})$$

Numerical solutions: Leggett mode

Weak dissipation

(a1, b1) Order parameters: **synchronization**



(c1) Decreasing particle number

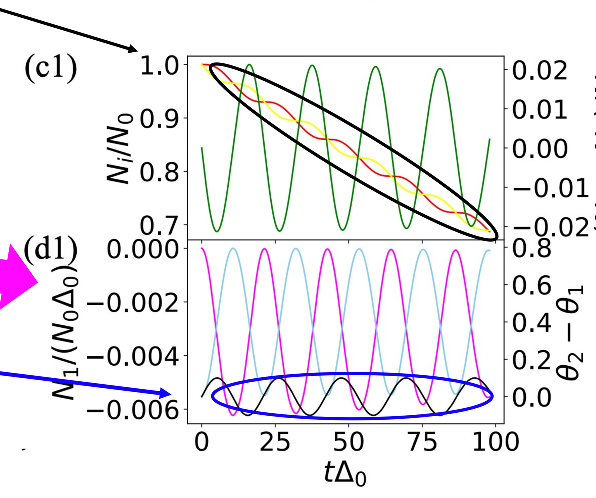
(d1) **Josephson current**

$$\frac{dN_1}{dt} = -\frac{4V|\Delta_1||\Delta_2|}{U_{\rm R}|U|^3}\sin(\theta_2 - \theta_1 + \delta)$$

the sign of current is negative (**finite dc component**)

Leggett mode

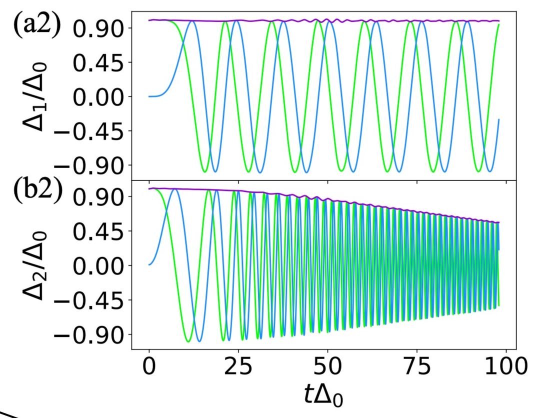
$$\omega_{\rm L}^2 = 4\left(\frac{\lambda_{12} + \lambda_{21}}{\det\lambda}\right)|\Delta_1||\Delta_2|$$



Nonequilibrium phase transition

Strong dissipation

(a2, b2) Order parameter: **faster phase rotation** in system 2



(c2) Particle number: much faster decreasing in system 2

system 1: **constant in the long-time limit**

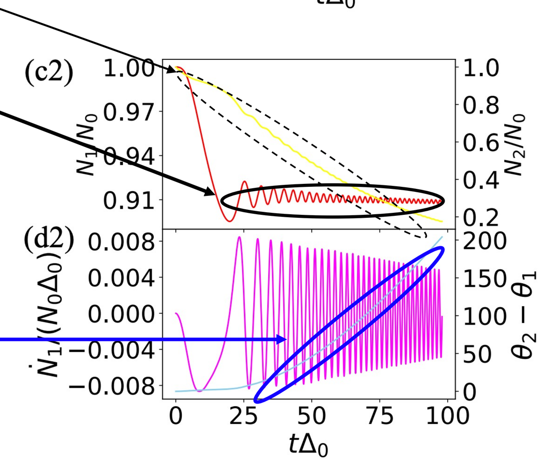
Continuous quantum Zeno effect

Effective loss rate $\gamma_{\rm eff} \to 0$

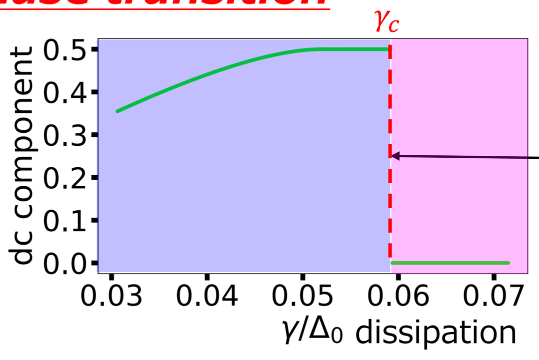
$\gamma_{\rm eff} = |V_{\rm eff}|^2/\gamma$, $V_{\rm eff} = V\Delta_2/U_{\rm R}$

(d) **Monotonic increase** in the phase difference

⇒ dc **Josephson current** vanishes



Dissipative dynamical phase transition



dc Josephson current vanishes !

$C = 1/2W$ $F = \gamma|\Delta_2|^2/|U|^2$ $R = +\infty$

Equation of motion

$$C\frac{d^2\Delta\theta}{dt^2} + \frac{1}{R}\frac{d\Delta\theta}{dt} + I_0\sin(\Delta\theta) = F$$

$$V_{\rm wash} = -2WI_0\cos(\Delta\theta) - \frac{2\gamma W|\Delta_2|^2\Delta\theta}{|U|^2}$$

$\gamma < \gamma_c$ Trapped in $V_{\rm wash}$

$\gamma > \gamma_c$ Running state

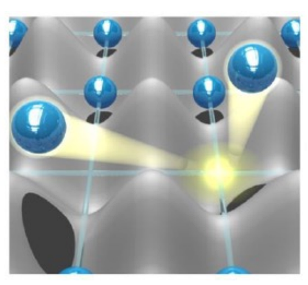
4. Discussions

Towards experimental realization

Candidate ?

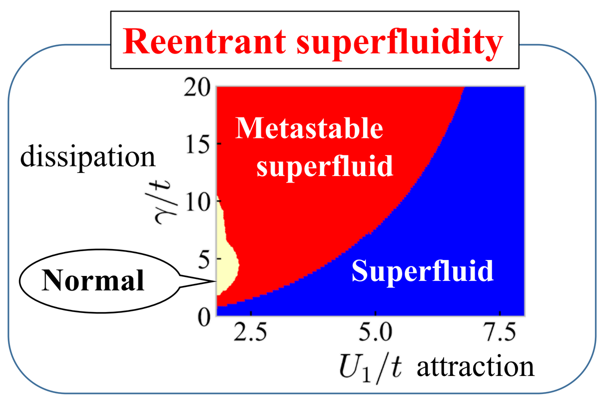
Ultracold atoms

Dissipation ⇒ **Photoassociation** techniques

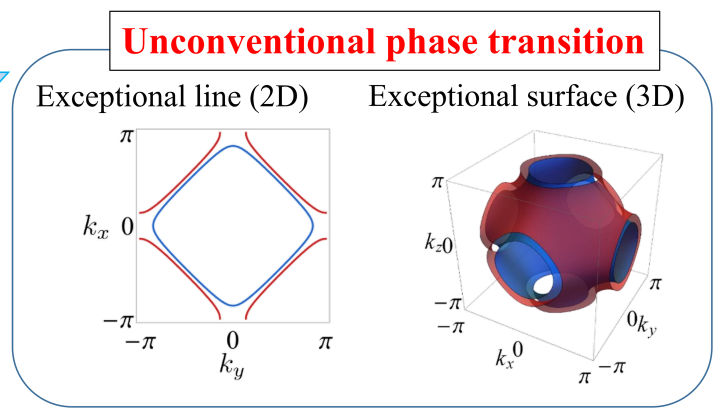


(Our) related work: ***Non-Hermitian fermionic superfluidity***

Reentrant superfluidity



Unconventional phase transition



K. Yamamoto *et al.*, PRL 123, 123601 (2019).