Stochastic Schrödinger Equations for Qubit Evolution

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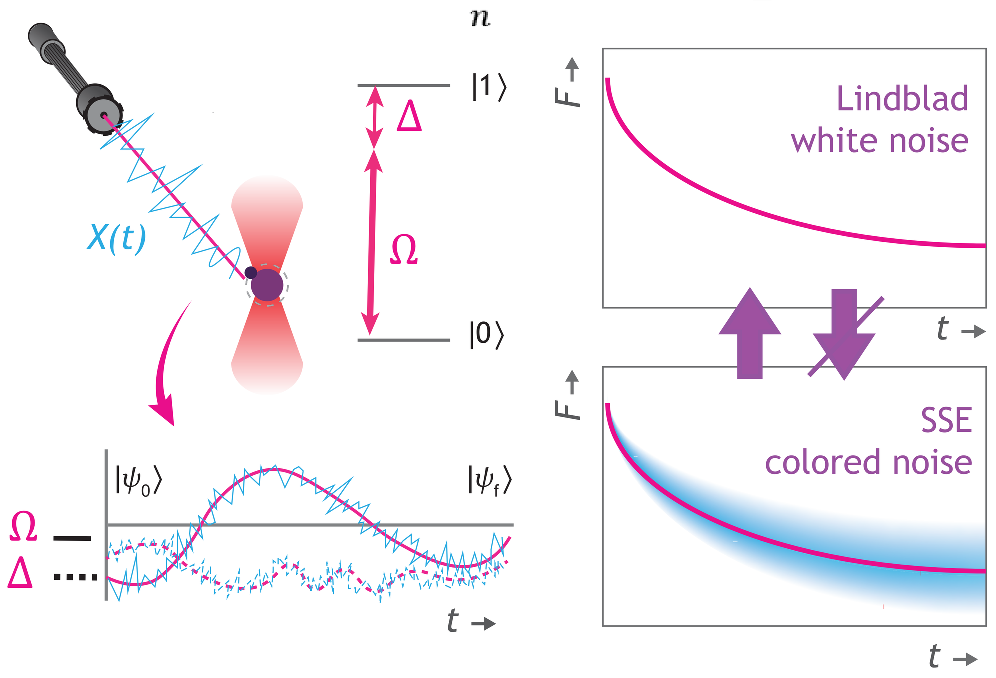
NISQ era 1

- **N**oisy **I**ntermediate **S**cale **Q**uantum
- Precursor of error-correction
- Important to understand noisy sources
- Noise described by Lindblad equation

$$\partial_t\rho = -i[H,\rho] + \sum_n \gamma_n S_n \rho S_n^\dagger - \frac{\gamma_n}{2}\{S_n^\dagger S_n, \rho\}$$

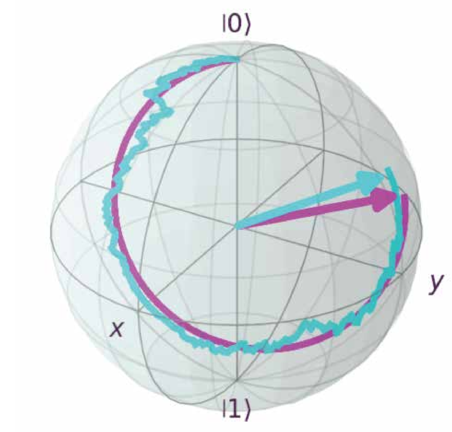


SSE (Stochastic Schrödinger Equation) 2

$$d\psi = -iH\psi dt - \frac{1}{2}S^\dagger S\psi dX^2 + iS\psi dX$$

$$d\phi = -iH\phi dt$$

Fidelity = $|\phi^\dagger\psi|^2$



- Norm preserving for $S = S^\dagger$
- Valid for semi-martingale X
- Full distribution wavefunctions
- Notoriously hard numerically:
  - Non-Euclidean space
  - Non-Linear noise

$$(WN):\ dX = \gamma dW_t$$

$$(OU):\ dX = -kXdt + \gamma dW_t$$

Pauli noise: $[H,S]=0$, $S^2 = I$ (e.g $S=\sigma_X$)

Projection noise: $[H,S]=0$, $S^2 = S$ (e.g $S=(I-\sigma_z)/2$)

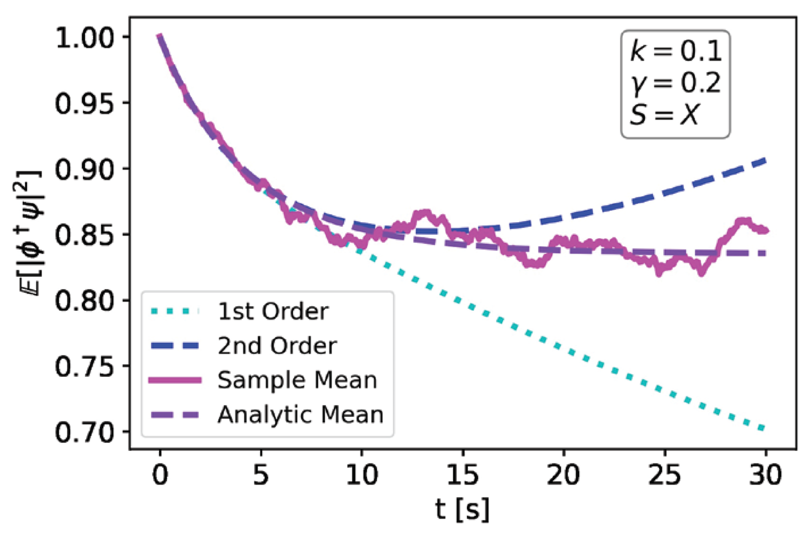
Solution Methods 3

Using Ito Calculus

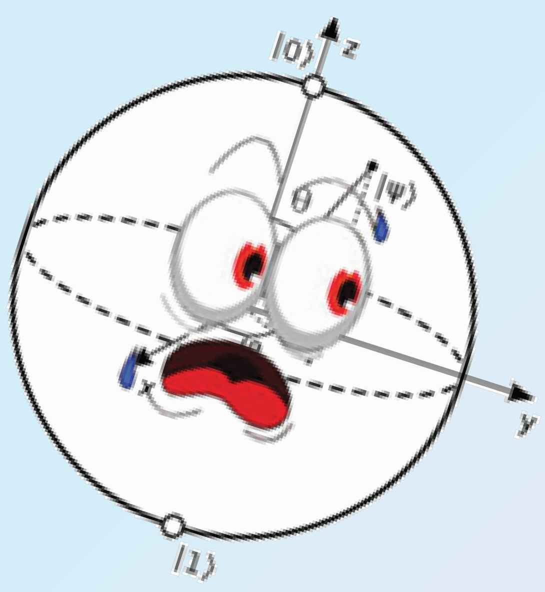
$$dx = Axdt + BxdX + adt + bdX$$

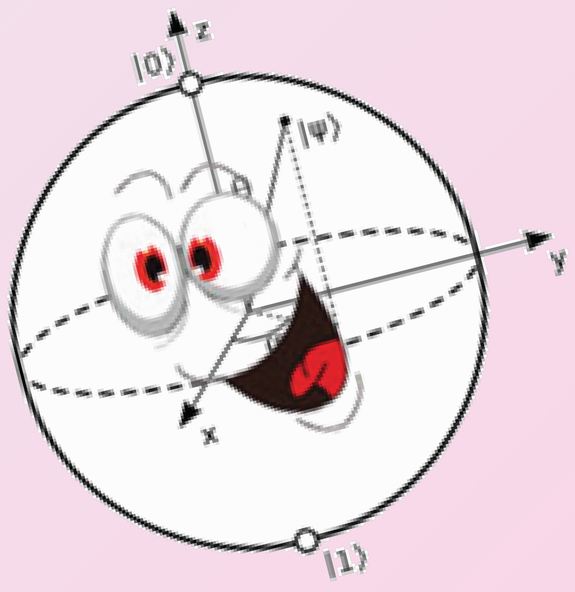
$$x_{(0)} = |\phi^\dagger\psi|^2$$

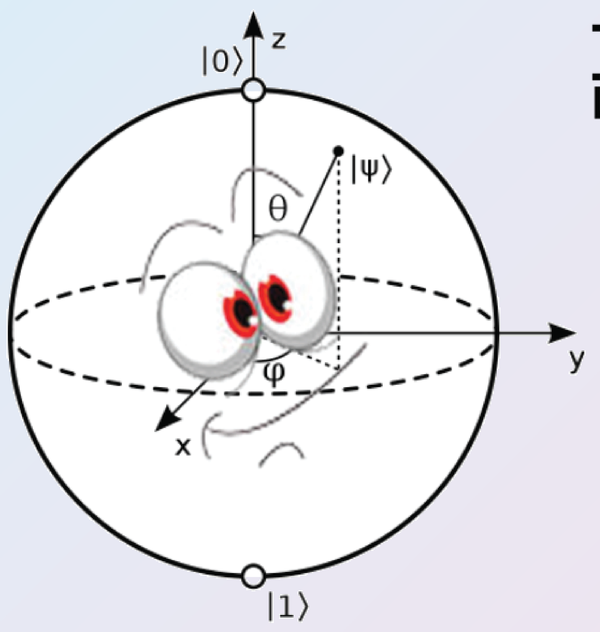
- First moment approximation order → Unphysical behavior
- Full stochastic solution (novel)

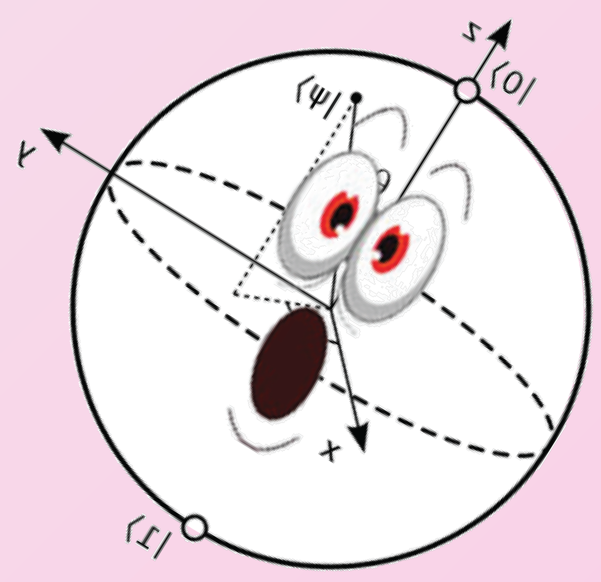


DO QUBITS LIKE METALLICA?





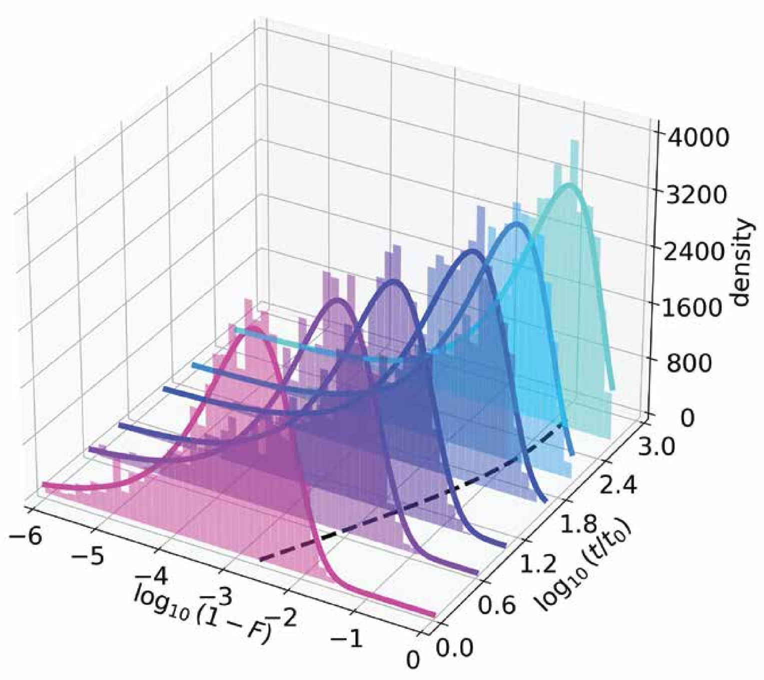




Distributions 4

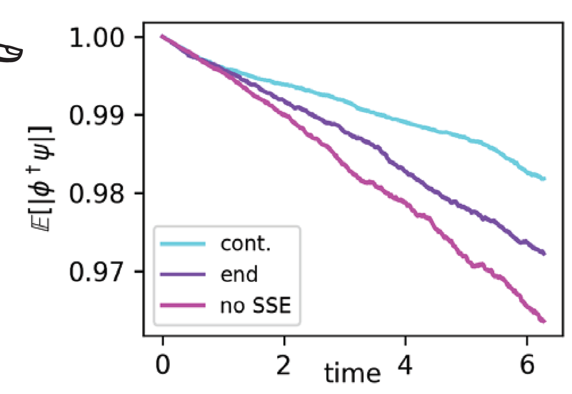
$$|\phi^\dagger\psi|^2 = \frac{1}{2}(1+S_0) + \frac{1}{2}(1-S_0)\cos(2(X_t - X_0))$$

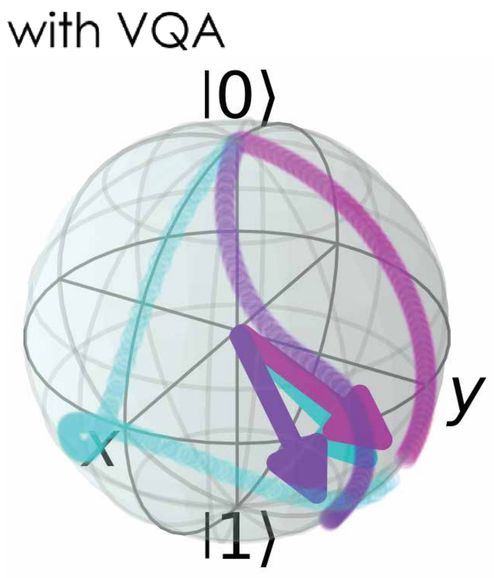
- More information than density matrices
- Calculate arbitrary moments
- Important for optimal control



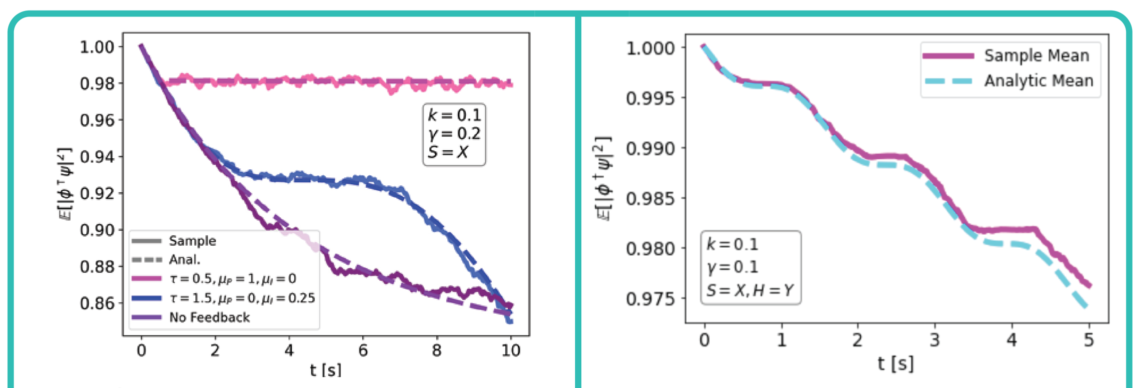
Optimal Control 5

- Mitigate movement through noisy areas
- Stochastic optimal control
- Adjoint-like method
- In combination with VQA





Ongoing work 6



$$dY_t = \begin{cases} dX_t & t<\tau \\ dX_t - \mu_P dX_{t-\tau} - \mu_I X_{t-\tau}dt & t \geq \tau \end{cases}$$

$$H = \alpha P_1, S = P_2, [H,S] = 2i\alpha P_3$$

Extending the methods:

- Non-commuting Hamiltonians
- General PSDs (see Poster P32)
- Multiqubit generalization
- Noise Mitigation

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