Staggered Superfluidity of Ultracold Bosons at Finite Temperatures

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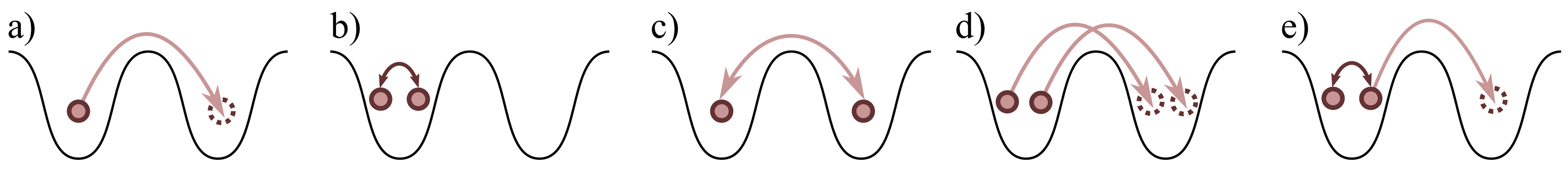
Non-standard Bose-Hubbard model

- The model Hamiltonian for spinless dipolar bosons in two-dimensional square lattice is

$$\hat{H} = -t\sum_{\langle i,j\rangle}\left(\hat{b}_i^\dagger\hat{b}_j + \text{H.c.}\right) + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) + \sum_{\langle i,j\rangle}\left[V\,\hat{n}_i\hat{n}_j - t'\,\hat{b}_i^\dagger(\hat{n}_i + \hat{n}_j)\hat{b}_j\right] - \mu\sum_i \hat{n}_i.$$

- $\hat{b}_i^\dagger$ ($\hat{b}_i$) is bosonic creation (annihilation) operator at ith lattice site, and $\hat{n}_i = \hat{b}_i^\dagger\hat{b}_i$ is the corresponding number operator.



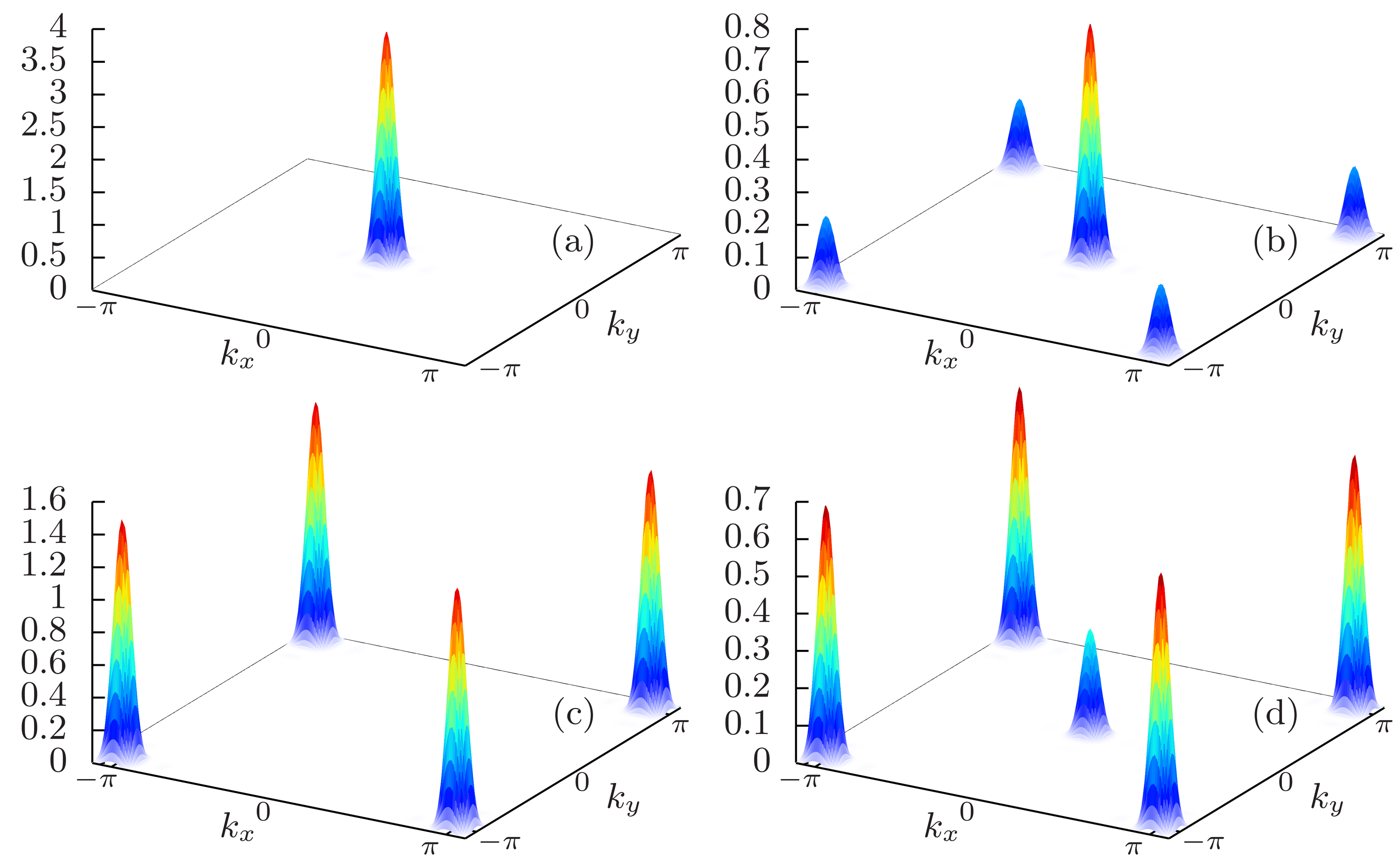
Model parameters:

- t: Hopping strength of atoms between nearest-neighbor sites.
- U: On-site repulsive interaction strength between atoms.
- V: Nearest-neighbour (NN) off-site repulsive interaction strength.
- t': Strength of density-induced tunneling.
- μ: Chemical potential for a fixed total number of particles.
- The many-body Gutzwiller wave function is

$$|\Psi_{\text{GW}}\rangle = \prod_i |\psi_i\rangle = \prod_i \sum_n^{n_{\max}} c_n^i |n_i\rangle.$$

- Finite-temperature phase transitions are investigated using mean-field Gutzwiller ansatz and quantum Monte Carlo approaches.

Momentum distributions of single-particle correlation function



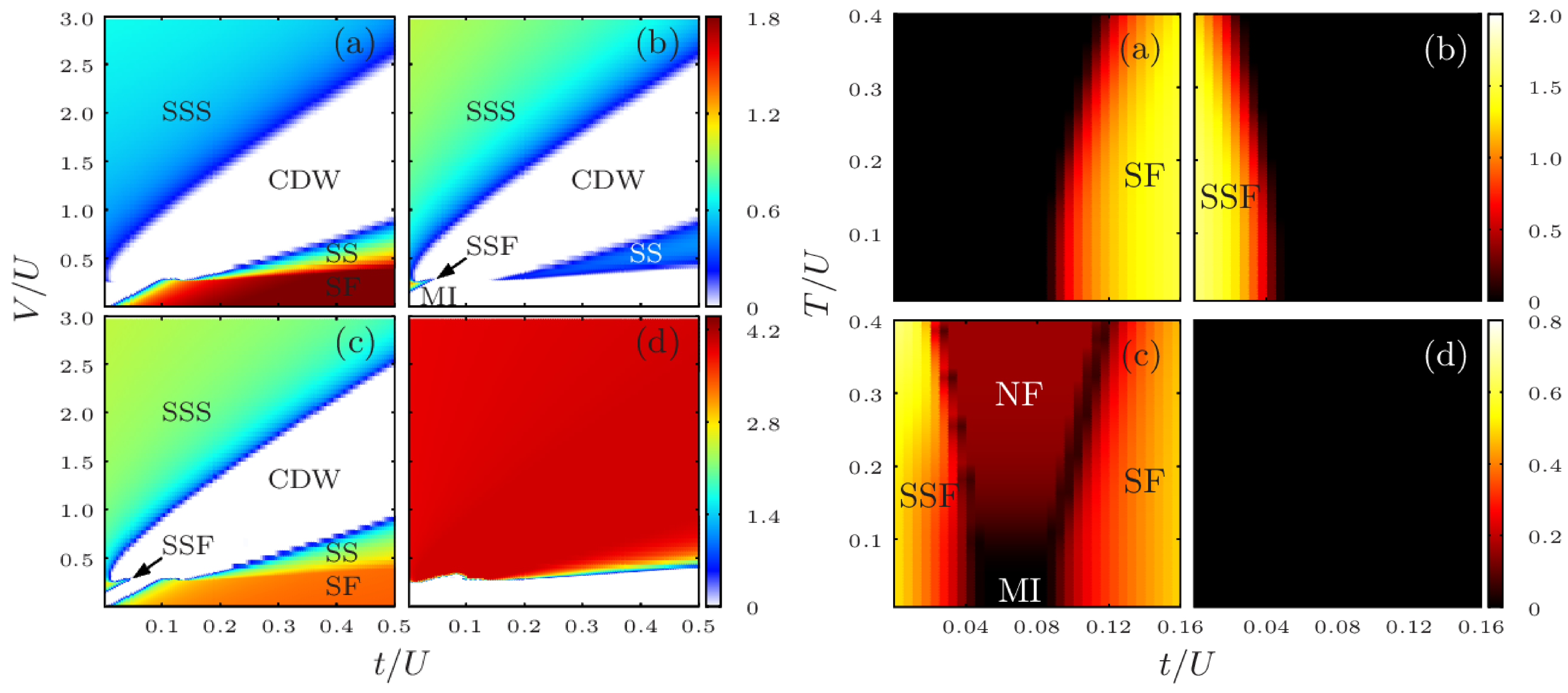
- The single-particle correlation and structure factor, measure of the off-diagonal and diagonal long-range order are

$$M(\mathbf{k}) = \frac{1}{L^2}\sum_{j,j'} e^{i\mathbf{k}\cdot(\mathbf{r}_j - \mathbf{r}_{j'})}\langle\hat{b}_j^\dagger\hat{b}_{j'}\rangle,$$

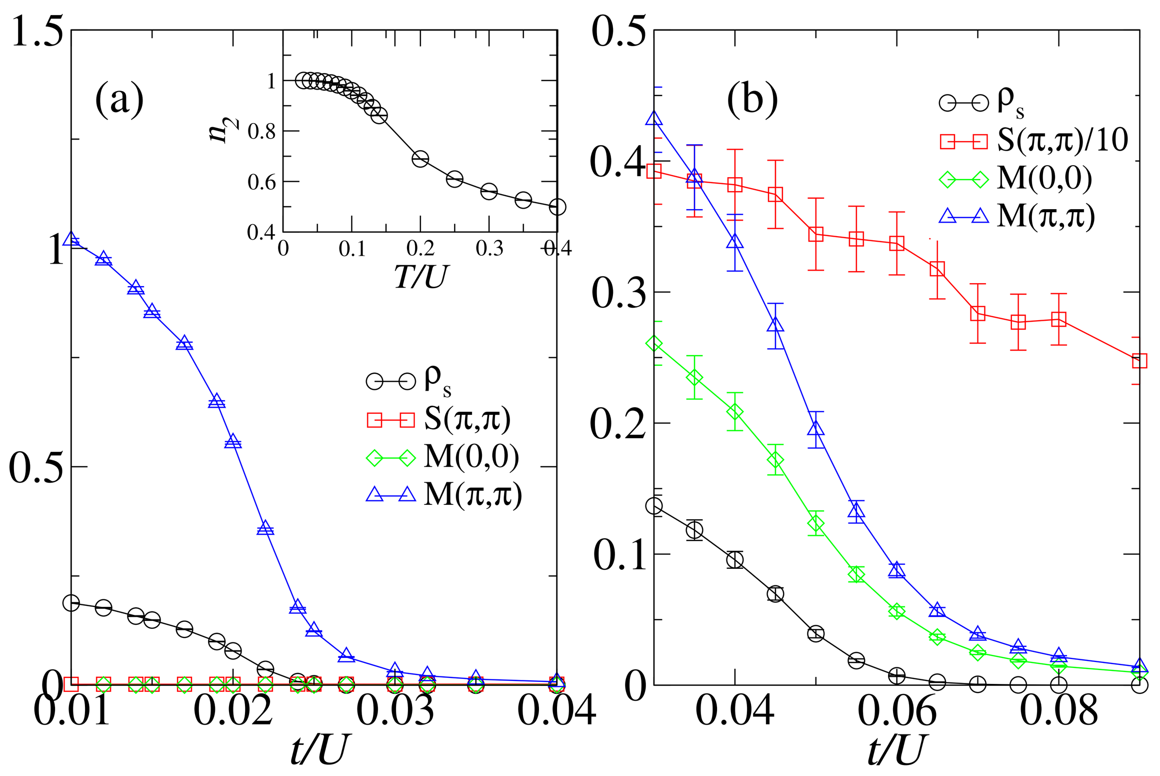
$$S(\mathbf{k}) = \frac{1}{L^2}\sum_{j,j'} e^{i\mathbf{k}\cdot(\mathbf{r}_j - \mathbf{r}_{j'})}\langle\hat{n}_j\hat{n}_{j'}\rangle.$$

- Various quantum phases of non-standard Hubbard model are
  - (a) **SF phase**: the off-diagonal long-range order associated with superfluidity results in a sharp peak in $M(\mathbf{k})$ at the center of the Brillouin zone.
  - (b) **SS phase**: the nearest-neighbor interaction leads to a compressible supersolid phase which exhibits a sharp peak at the center and smaller peaks at the corners of the Brillouin zone.
  - (c) **SSF** and (d) **SSS phases**: for both phases the largest peaks move to the corners of the Brillouin zone.

Phase diagrams at zero and finite T

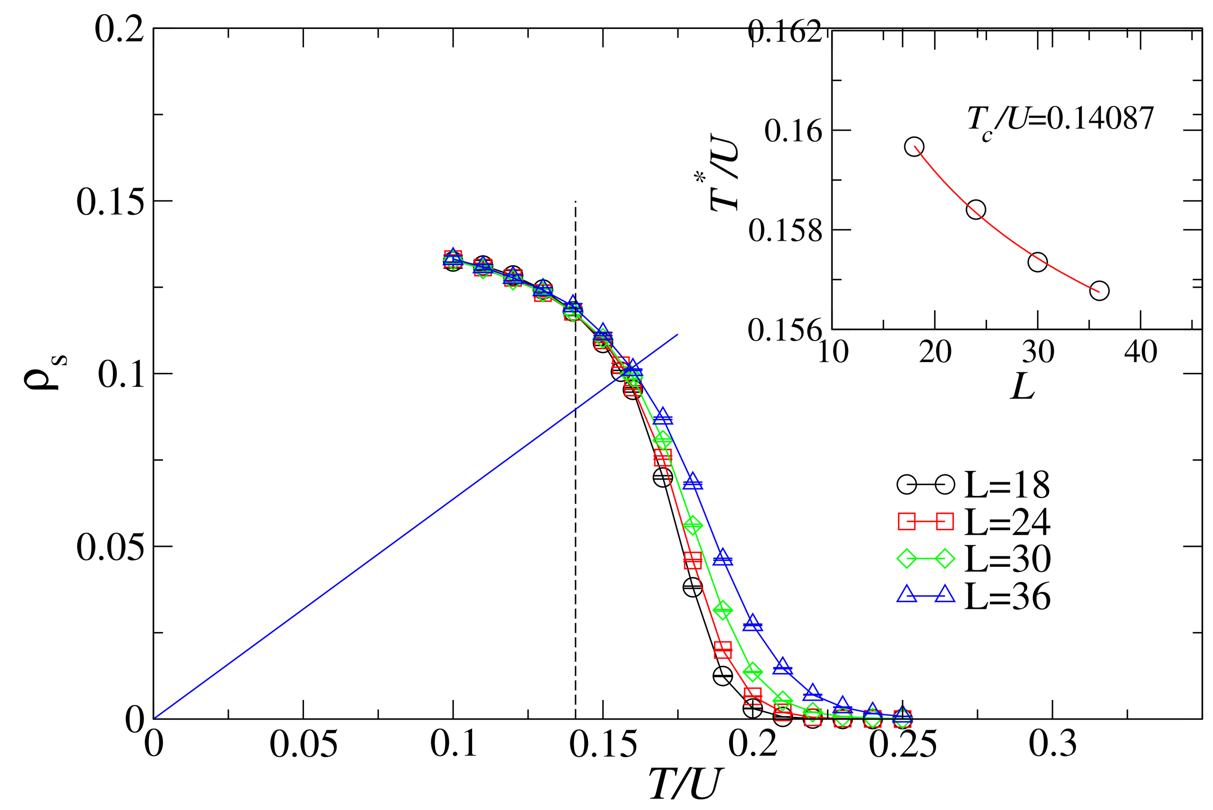


- Previous studies show the existence of SSF due to destructive interference between correlated and single-particle hopping.
- At low (higher) temperatures, two topologically distinct superfluids are separated by MI (normal) state.
- The T_c of SSF decreases while that of SF increases with t.



- Order parameters at (a) weak $V = 0.24$ and (b) strong $V = 1$ NN interactions.

Finite-size scaling



- The blue line is superfluidity $\rho_s = (2/\pi)T$, and its intersection with the $\rho_s(T)$ for various L gives $T^*(L)$.
- The inset figure shows the logarithmic dependence expected for Kosterlitz-Thouless (KT) transition.
- Staggered ordering of the superfluidity does not alter the nature of the thermal phase transition.

Summary

- We present a parameter regime for the existence of staggered superfluidity, following KT-type thermal phase transition.
- This study paves a way to realize staggered quantum phases in dipolar cold-atom experiments.

References

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