# Port-based Entanglement Teleportation with Noisy Resource State

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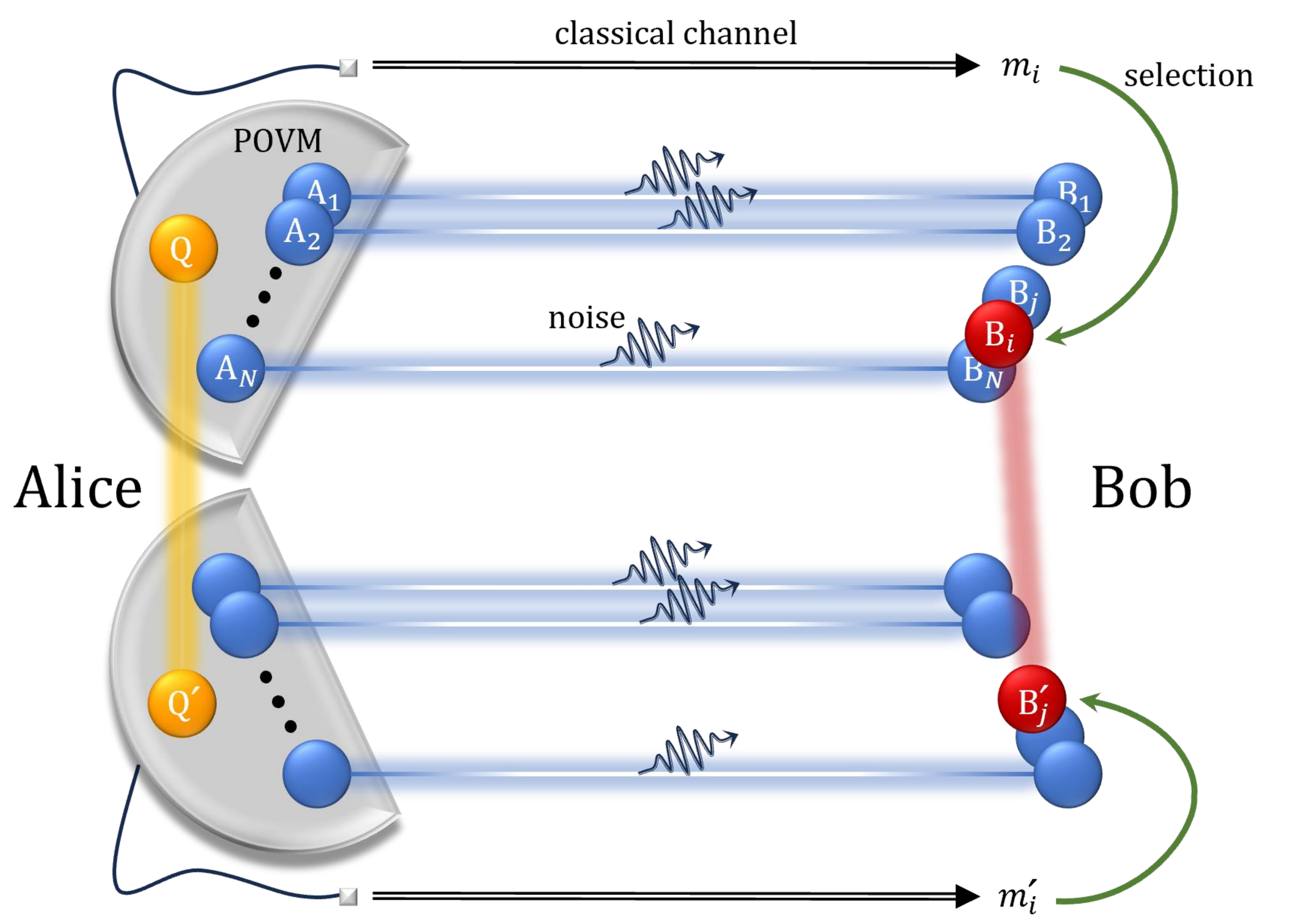
## Abstract

- We consider port-based teleportation[1](PBT) with resource states influenced by Pauli noise.
- We find that the channel of noisy PBT can be represented as a chain of channels with number of ports and environment noise.
- We derive that the order of entanglement is preserved under Pauli channels.
- We investigate the upper and lower bound of the entanglement teleportation[2].

## Model : Port-based Teleportation




- Sending unknown bipartite two qubits state using PBT to each qubit separately.
- Pauli noise affecting each qubit of the resource states:

$$\mathcal{E}_{\vec{p}}(\hat{\rho}) = \frac{p_0}{4}\hat{\rho} + \frac{p_1}{4}\hat{X}\hat{\rho}\hat{X} + \frac{p_2}{4}\hat{Y}\hat{\rho}\hat{Y} + \frac{p_3}{4}\hat{Z}\hat{\rho}\hat{Z}$$

- Average of channel(noise) probabilities

$$\Omega = \frac{1}{3}(p_1 + p_2 + p_3)$$

- Positive partial transpose criterion as measure of entanglement : $\mathcal{M}(\hat{\rho})$
- Entanglement of initial unknown state:

$$\mathcal{M}_0 = \sin\theta$$

## Result : Channel Decomposition

- Single noisy PBT channel $\Lambda$ can be represented as a chain of channels with teleportation fidelity of noiseless PBT $f(\Lambda_0)$ and bilinear function of noise probabilities $\vec{p}$ caused by environment:

$$\mathrm{Tr}_{Q\overrightarrow{AB_i}}\left[\left(\hat{\Pi}^{(i)}_{\mathrm{POVM}}\otimes\hat{I}\right)\cdot\left(\hat{\rho}\otimes\tilde{\mathcal{E}}\left[|\Psi^+\rangle\langle\Psi^+|\right]^{\otimes 2N}\right)\right] = \left[\mathcal{E}^{\mathrm{dep}}_{2(1-f(\Lambda_0))}\circ\mathcal{E}_{\vec{B}(\vec{p})}\right]\hat{\rho}$$

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## References

[1] S. Ishizaka, T. Hiroshima, Phys. Rev. Lett. 101,240501 (2008).
[2] J. Lee, M. S. Kim, Phys. Rev. Lett. 84, 4236-4239 (2000).

## Result : Entanglement Order Preserving

❖ Consider $\mathcal{E}_{\vec{p}}$ and $\mathcal{E}_{\vec{p}'}$ as Pauli channels acting on a single qubit. If the entanglement of bipartite mixed sates $\hat{\rho}_{\mathrm{A}}$ and $\hat{\rho}_{\mathrm{B}}$ satisfies

$$\mathcal{M}(\hat{\rho}_{\mathrm{A}}) > \mathcal{M}(\hat{\rho}_{\mathrm{B}}) > 0,$$

then it follows that

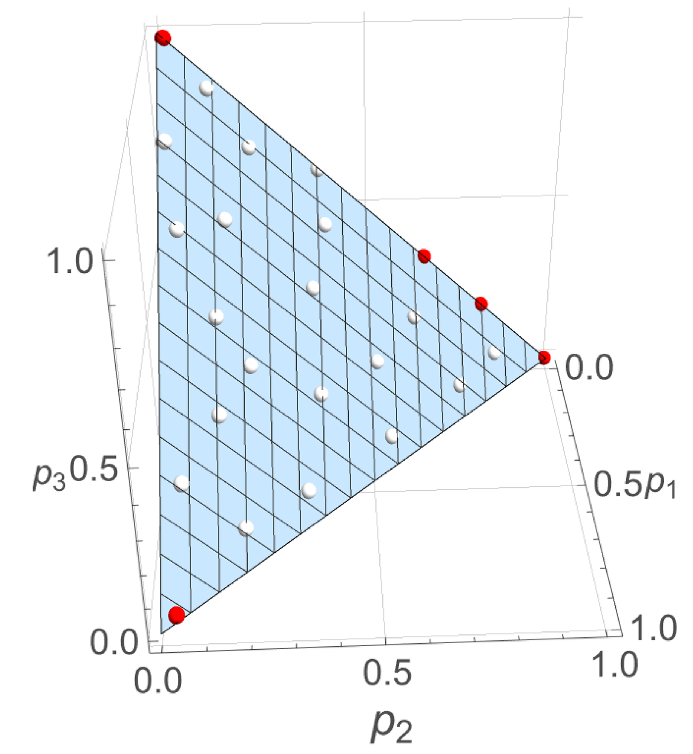
$$\mathcal{M}\left(\left[\mathcal{E}_{\vec{p}}\circ\mathcal{E}_{\vec{p}'}\right]\hat{\rho}_{\mathrm{A}}\right) > \mathcal{M}\left(\left[\mathcal{E}_{\vec{p}}\circ\mathcal{E}_{\vec{p}'}\right]\hat{\rho}_{\mathrm{B}}\right)$$

for all $\mathcal{E}_{\vec{p}}$ and $\mathcal{E}_{\vec{p}'}$ that satisfies

$$\mathcal{M}\left(\left[\mathcal{E}_{\vec{p}}\circ\mathcal{E}_{\vec{p}'}\right]\hat{\rho}_{\mathrm{A}}\right) > 0.$$

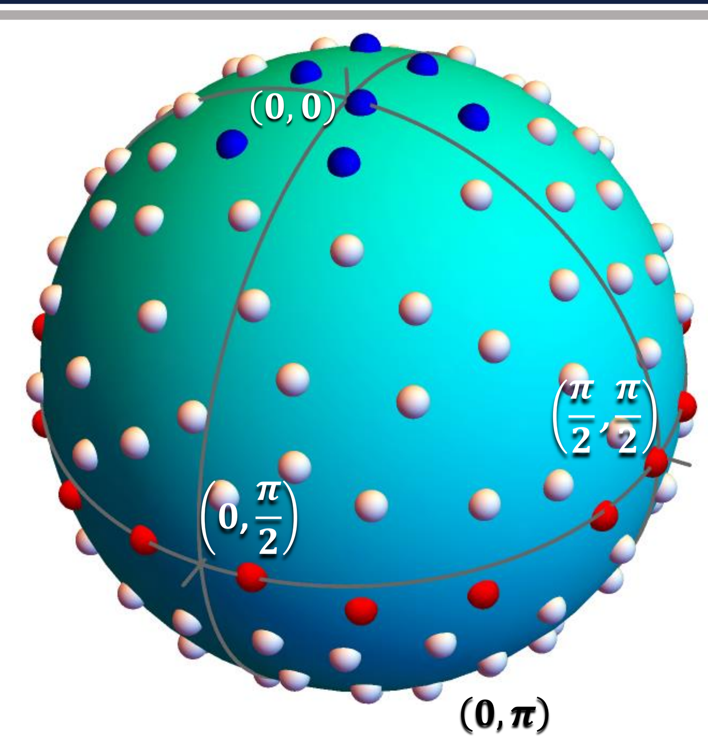
- The states teleported by the noisy PBT channels that maximize or minimize the entanglement relative to the **average of channel probabilities** $\Omega$ and the **initial entanglement** $\mathcal{M}_0$ are equivalent of the one on $\mathcal{E}_{\vec{B}(\vec{p})}(\hat{\rho})$.

## Result : Upper and Lower Bound




**Left :** The dots correspond to elements of a sample set of channel probabilities. Red dots are channels probabilities near lower and upper bound.

**Right :** The dots correspond to elements of a sample set of rotating angles $(\alpha_1, \alpha_2)$ and $(\beta_1, \beta_2)$. Red dots are angles near upper bound and blue dots are ones near lower bound through phase flip channel.

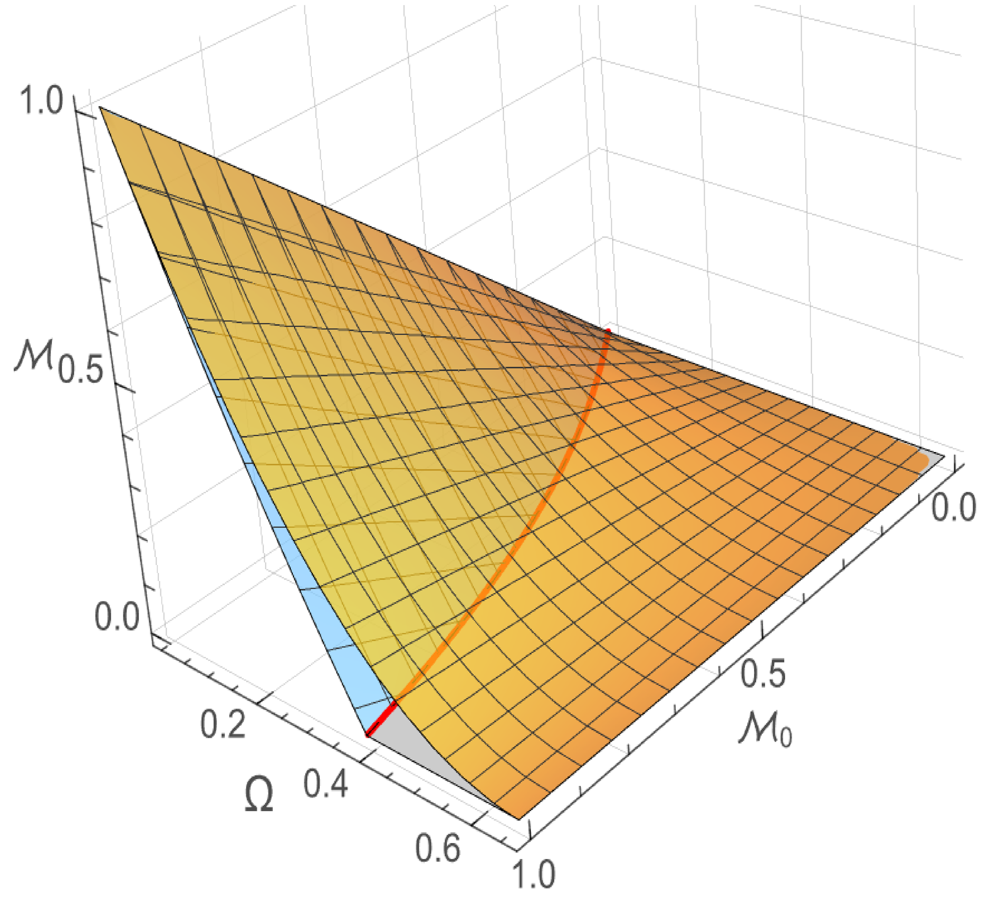


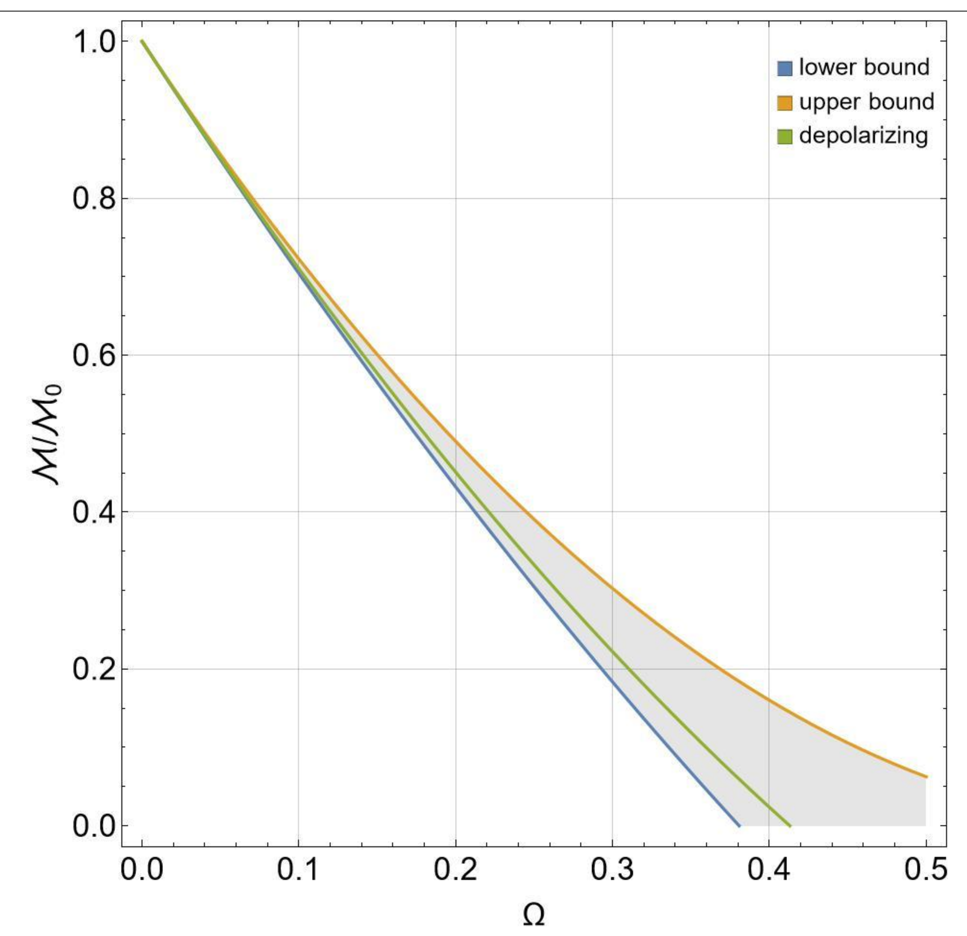

- Arbitrary two qubits state can be represented as

$$\left[\left(\hat{R}_Z(\alpha_1)\cdot\hat{R}_Y(\alpha_2)\cdot\hat{R}_Z(\gamma)\right)\otimes\left(\hat{R}_Z(\beta_1)\cdot\hat{R}_Y(\beta_2)\right)\right]\cdot\left[\cos\left(\frac{\theta}{2}\right)|0\rangle|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle|1\rangle\right]$$

- Regardless of $\Omega$ and $\mathcal{M}_0$, the channels of boundaries are bit, bit-phase, phase flip channels.
- For phase flip channels, the initial states of upper and lower bound are

$$|U\rangle = \cos\left(\frac{\theta}{2}\right)|+\rangle|+\rangle + i\sin\left(\frac{\theta}{2}\right)|-\rangle|-\rangle, \qquad |L\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle|0\rangle + \sin\left(\frac{\theta}{2}\right)|1\rangle|1\rangle.$$




**Left :** Lower and upper bound of entanglement. **Right :** Bounds of relative entanglement at $\mathcal{M}_0 = 0.8$.

- Entanglement of the unknown state teleported with noisy PBT is

$$\mathcal{M} \approx \mathcal{M}_0 - (\mathcal{M}_0\alpha + \beta)\Omega - (2\mathcal{M}_0 - 1)\frac{1}{N}$$

bounded with $0 \le \beta \le \frac{1}{2}, 0 \le \alpha + \beta \le 1$ at weak noise and large number of ports.

## Conclusions

- Errors due to the noise of environment on resource states and limitation of size of ports can be handled separately by using a chain of two independent channels.
- The order of entanglement of two qubits states is not invariant to the separated Pauli channels.
- The lost of entanglement is composed of a term proportional to the average of noise probabilities with a slope of 0 to 1, and a term reciprocal to the number of ports with a slope of 0 to 1.

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