Thermal Decay of Planar Jones-Roberts Solitons

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Abstract

Finite temperature effects on structures in quantum many body systems are still not well understood. To account for thermal effects, we employ the Stochastic Projected Gross Pitaevskii equation (SPGPE). This equations adds two damping terms, so called number damping and number conserving energy damping. We find bounds for the dominance of the two mechanisms in the case of a Jones-Roberts Soliton. These bounds suggest that for most experiments energy damping is the relevant process. As the damping terms yield qualitatively different decay, our work provides a testbed for current thermal theory of Bose-Einstein condensates.

Bosonic Quantum Many-Body Systems

Hamiltonian for bosonic quantum many-body systems:

$$\hat{H} = \int d^3\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})H_{\rm sp}\hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d^3\mathbf{r}\int d^3\mathbf{r}'\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')\mathcal{W}(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r})$$

$H_{\rm sp}$: Single particle Hamiltonian
$\mathcal{W}(\mathbf{r}-\mathbf{r}')$: Interaction potential

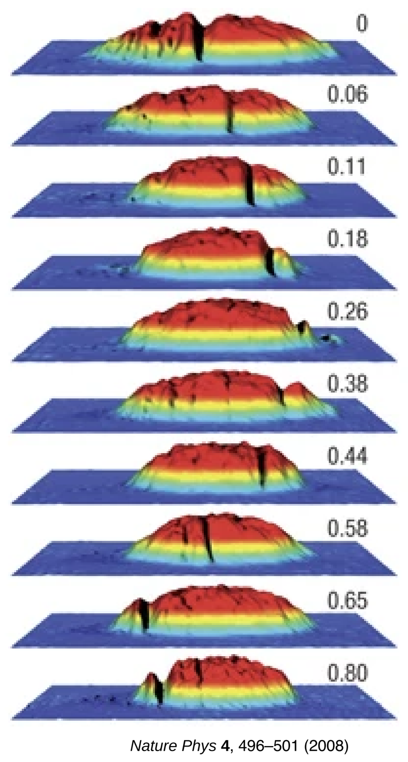
low temperature limit: most of the atoms built a Bose-Einstein Condensate (BEC) evolving as a single particle wave-function according to the Gross-Pitaevskii equation

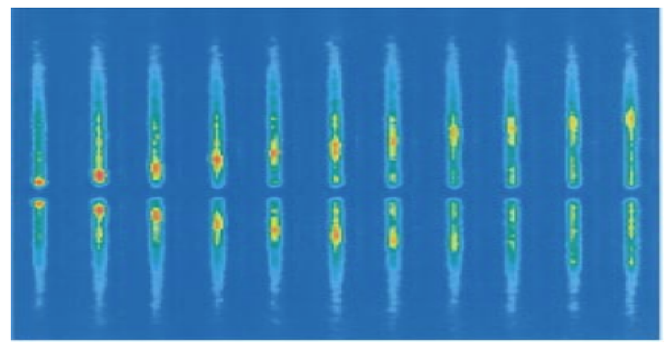
$$i\hbar\partial_t\psi(\mathbf{r}) = H_{\rm sp}\psi(\mathbf{r}) + g|\psi(\mathbf{r})|^2\psi(\mathbf{r}),\quad g = \frac{4\pi\hbar^2 a_s}{m}$$

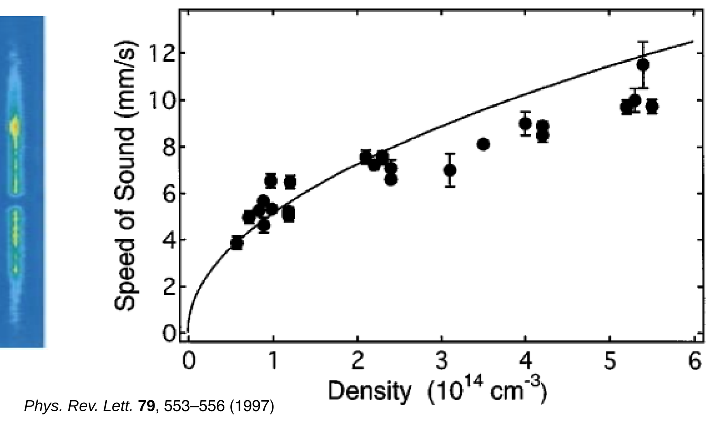
a_s : s-wave scattering length

Structures in Low Dimensional BEC

Sound with speed $c = \sqrt{\frac{n_0 g}{m}}$







Dark Soliton

- Form preserving evolution in 1D with sub-sonic velocity v
- Phase jump over the soliton
- Wides and flattens with increasing velocity
- Analytical form:

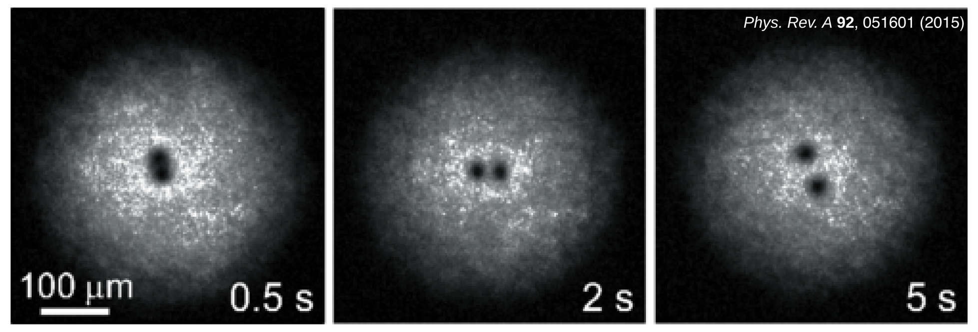
$$\psi(x) = \sqrt{n_0}\left[i\frac{v}{c} + \epsilon\tanh\left(\frac{\epsilon x}{\sqrt{2}\xi}\right)\right]$$

Healing length $\xi = \frac{\hbar}{\sqrt{2mn_0g}}$

$$\epsilon = \sqrt{1-(v/c)^2}$$

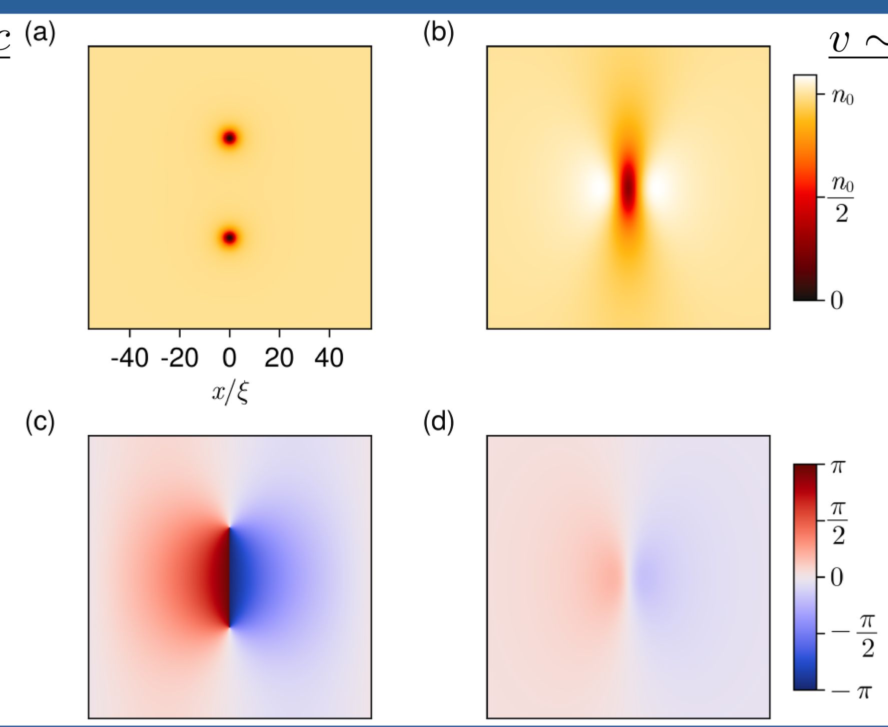
Vortices

- Only vortices of uniform size
- Density vanishes at position of vortex
- Phase changes on closed loops around a vortex by an integer multiple of 2π

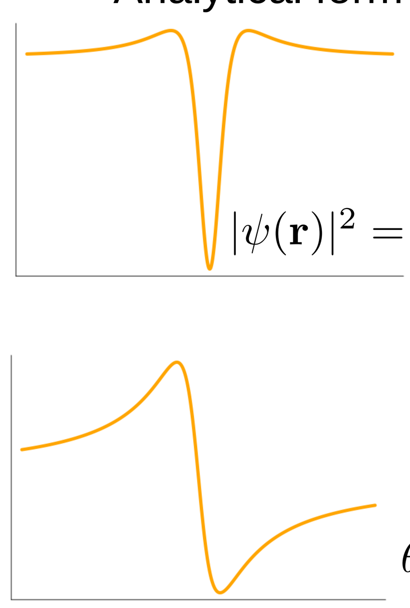


Jones-Roberts Soliton

- Self similar evolution in 2D
- Travels with sub-sonic velocity
- Widens and flattens with increasing velocity
- Robust to small disturbances



Analytical form close to the speed of sound[2]:

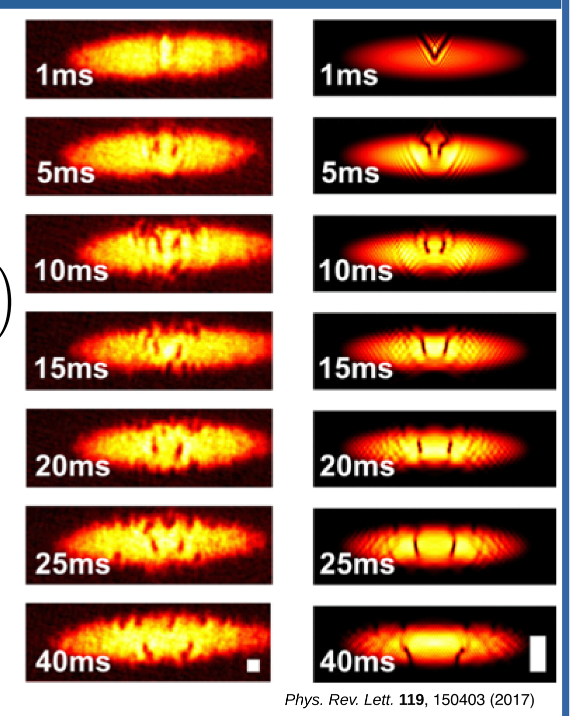


Density

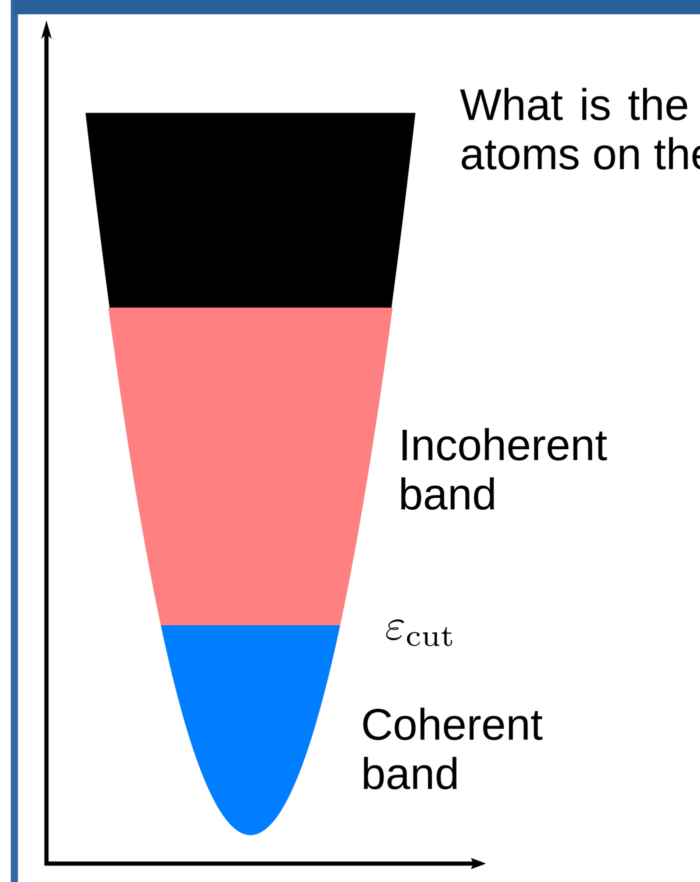
$$|\psi(\mathbf{r})|^2 = n_0\left(1 - 4\epsilon^2\frac{3/2+\epsilon^4y^2/\xi^2-\epsilon^2x^2/\xi^2}{[3/2+\epsilon^4y^2/\xi^2+\epsilon^2x^2/\xi^2]^2}\right)$$

Phase

$$\theta(\mathbf{r}) = -2\sqrt{2}\epsilon\frac{\epsilon x/\xi}{3/2+\epsilon^4y^2/\xi^2+\epsilon^2x^2/\xi^2}$$



Finite Temperature Effects



What is the influence of non condensed atoms on the dynamics of the BEC?

$$d\psi|_H = \mathcal{P}\left\{-\frac{i}{\hbar}(H_{\rm sp}+g|\psi|^2-\mu)\psi dt\right\},$$

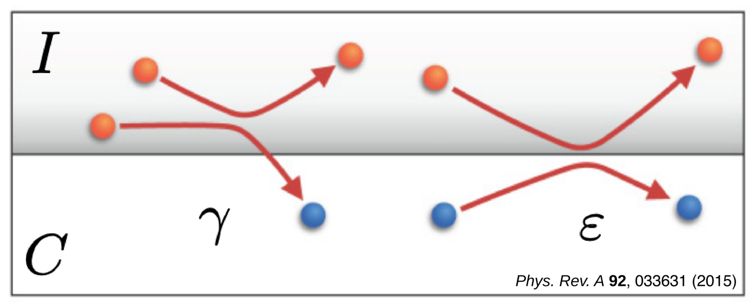
Describing the Hamiltonian evolution ($\mathcal{P}$: projection on the coherent band)

$$d\psi|_\gamma = \mathcal{P}\left\{-\frac{\gamma}{\hbar}(H_{\rm sp}+g|\psi|^2-\mu)\psi dt + dW(\mathbf{r},t)\right\},\quad (\mathbf{S})d\psi|_\varepsilon = \mathcal{P}\left\{-\frac{i}{\hbar}V_\varepsilon(\mathbf{r},t)\psi dt + i\psi dU(\mathbf{r},t)\right\}$$

- To account for this interaction: include an incoherent band
- Contains atoms with energy $> \varepsilon_{\rm cut}$
- Atoms in this band assumed to be thermally distributed
- Stochastic projected Gross-Pitaevskii equation (SPGPE)

$$(\mathbf{S})d\psi = d\psi|_H + d\psi|_\gamma + (\mathbf{S})d\psi|_\varepsilon$$ [3]

dW : Complex noise
γ : Damping rate



dU : Real noise
V_ε : Scattering potential (ψ-dependent)

Describing **number damping**, scattering between non-condensate band atoms leading to growth of the condensate band.

Describing **energy damping**, scattering between non-condensate band atoms and atoms in the condensate band without particle exchange.

Damping of Jones-Roberts Solitons

Renormalized Lagrangian $L = \int d^2\mathbf{r}\left[\frac{i\hbar}{2}(\psi^*\partial_t\psi - \psi\partial_t\psi^*)\left(1-\frac{n_0}{|\psi|^2}\right) - \frac{\hbar^2}{2m}|\nabla\psi|^2 - \underbrace{\frac{g}{2}(|\psi|^2-n_0)^2}_{E_{\rm int}}\right]$

Perturbed Euler-Lagrange equation

Gross-Pitaevskii equation with small disturbance $i\hbar\partial_t\psi(\mathbf{r}) = -\frac{\hbar^2}{2m}\Delta\psi(\mathbf{r}) + g(|\psi(\mathbf{r})|^2 - n_0)\psi(\mathbf{r}) + \mathcal{U}\psi$

Leads for JRS Solution to $\frac{\partial L}{\partial x_s} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_s} = 2\Re\left\{\int d^2\mathbf{r}\,(\mathcal{U}\psi)^*\frac{\partial\psi}{\partial x_s}\right\}$, Momentum $P = \frac{\partial L}{\partial \dot{x}_s}$

Number damping: $\mathcal{U}\psi = -i\gamma\left[-\frac{\hbar^2}{2m}\Delta + g(|\psi|^2-n_0)\right]\psi \Rightarrow \left.\frac{dP}{dt}\right|_\gamma = -2\frac{\gamma m v}{\hbar}E_{\rm tot}$, Total energy $E_{\rm tot}$

Energy damping: $\mathcal{U}\psi = V_\varepsilon\psi = \frac{\hbar}{(2\pi)^2}\int d^2\mathbf{k}\tilde{\varepsilon}(\mathbf{k})\frac{d\tilde{n}(\mathbf{k})}{dt}e^{i\mathbf{k}\cdot\mathbf{r}}\psi(\mathbf{r}),\ \tilde{\varepsilon}(\mathbf{k}) = 8a_s^2N_{\rm cut}\exp\left(\frac{l_z^2\mathbf{k}^2}{4}\right)K_0\left(\frac{l_z^2\mathbf{k}^2}{4}\right)$

$N_{\rm cut}$: number of atoms in the lowest non-condensate state
l_z : thickness of the thermal atomic cloud

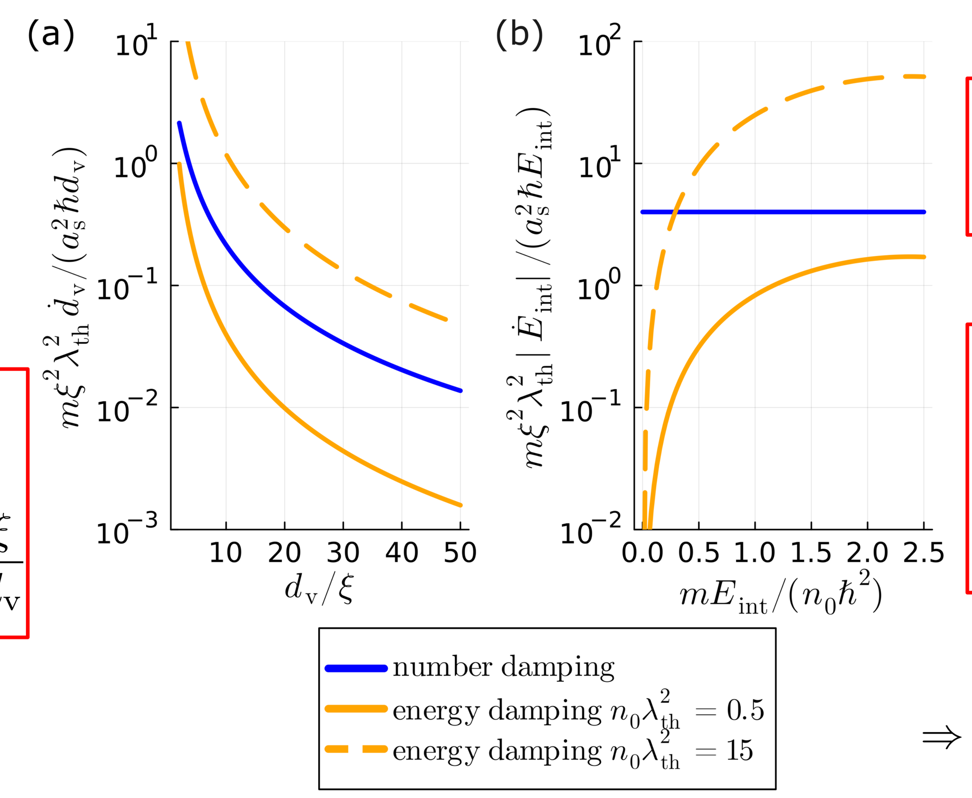
$$\Rightarrow \left.\frac{dP}{dt}\right|_\varepsilon = -\hbar v\int\frac{d^2\mathbf{k}}{(2\pi)^2}\tilde{\varepsilon}(\mathbf{k})\left|\widetilde{\partial_x n}(\mathbf{k})\right|^2$$

$v \ll c$ The momentum is given by the distance between the vortices d_v

$v \sim c$ The momentum is given by the Interaction energy $E_{\rm int} = \frac{4\pi}{3}\frac{n_0\hbar^2}{m}\epsilon$

$$\frac{m\xi^2}{\hbar}\frac{d}{dt}\frac{d_v}{\xi}\bigg|_\gamma = -2\gamma\frac{\xi}{d_v}\ln\left(1.46\frac{d_v}{\xi}\right)$$

$$\frac{m\xi^2}{\hbar}\frac{d}{dt}\frac{d_v}{\xi}\bigg|_\varepsilon = -4n_0a_s^2N_{\rm cut}F\left(\frac{l_z^2}{8\xi^2}\right)\frac{\xi}{d_v}$$



$$\frac{m\xi^2}{\hbar}\frac{d\epsilon}{dt}\bigg|_\gamma = -\gamma\epsilon$$

$$\frac{m\xi^2}{\hbar}\frac{d\epsilon}{dt}\bigg|_\varepsilon = \frac{32}{3}n_0a_s^2N_{\rm cut}\epsilon^3\left[2\ln\left(\epsilon\frac{l_z}{\xi}\right) - 0.56\right]$$

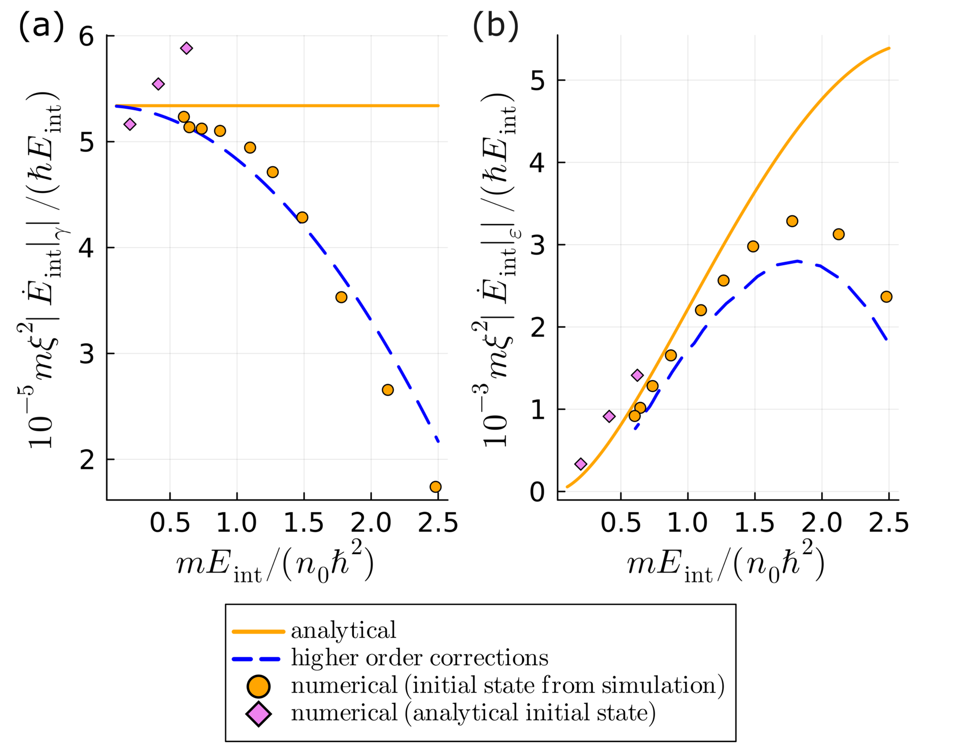
$\Rightarrow$ Qualitatively distinct damping!

$$8\frac{N_{\rm cut}^2}{(N_{\rm cut}+1)^2}\frac{a_s^2}{\lambda_{\rm th}^2} < \gamma < 8N_{\rm cut}\frac{a_s^2}{\lambda_{\rm th}^2}$$

$n_0\lambda_{\rm th}^2 < 1$: Number damping dominates

$n_0\lambda_{\rm th}^2 > 10$: Energy damping dominates

$\lambda_{\rm th} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$ Thermal de-Broglie wavelength



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[4] Mehdi, Z., Hope, J. J., Szigeti, S. S. & Bradley, A. S. Mutual friction and diffusion of two-dimensional quantum vortices. Phys. Rev. Res. 5, 013184 (2023).