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Designed Quantum States of Matter

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GHZ protocols enhance frequency metrology despite spontaneous decay

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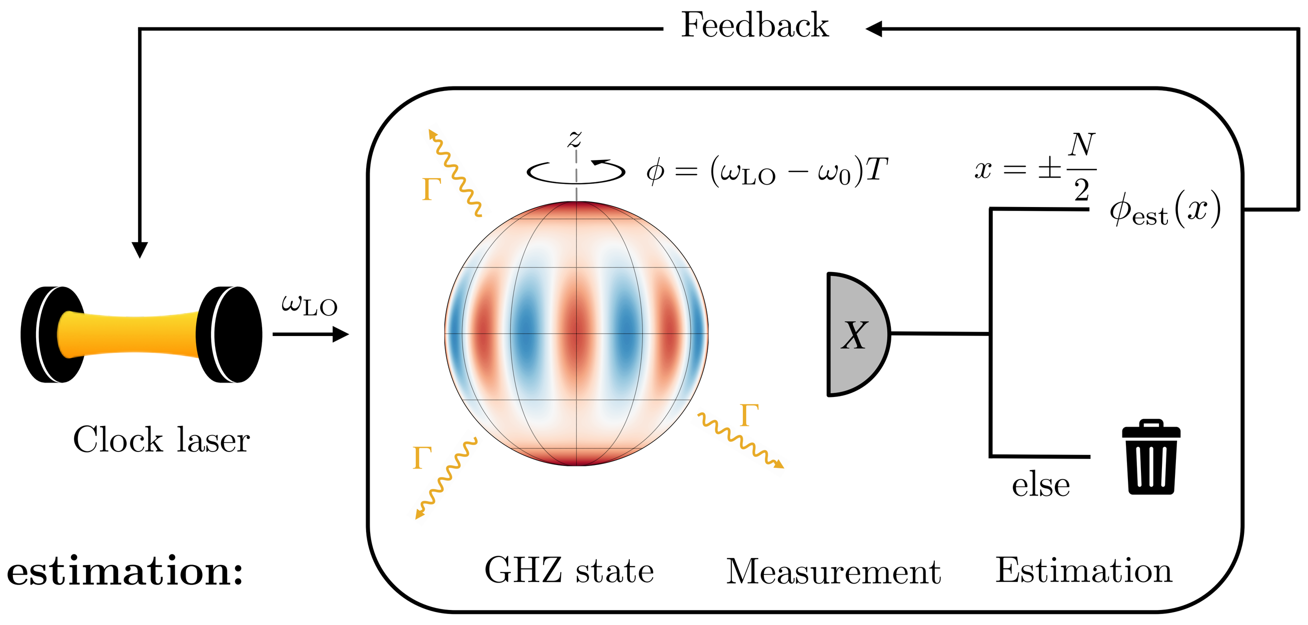
Introduction & Motivation

Optical atomic clocks are the most precise measurement devices achieving stabilities on the scale of 10^{-18} and below. This unprecedented performance opens the path towards new applications in fundamental research and technology, such as the redefinition of the SI second, relativistic geodesy, tests of general relativity and the search for physics beyond the standard model.

Current efforts to further improve the stability of optical clocks involve exploring the use of entanglement in atomic systems to reduce quantum projection noise and overcome the standard quantum limit (SQL) imposed by uncorrelated atoms. Unfortunately, decoherence presents a significant obstacle in frequency metrology, impairing the precision of measurements by compromising the coherence of quantum systems essential for achieving entanglement-based enhancement. In particular, the finite lifetime of the excited state represents a fundamental limit rather than an external noise, as magnetic field fluctuations or laser noise. With state-of-the-art clock lasers reaching coherence times of several seconds, the excited-state lifetime of several clock candidates becomes the more stringent limitation.

Frequency metrology



Phase estimation:

- local: phase ϕ is well centered around a fixed working point ϕ_0
- locally unbiased estimator ϕ_{est}

$$(\Delta\phi(T))^2 = \sum_x P(x|\phi_0, T)\,[\phi_{est}(x,T) - \phi_0]^2$$

Frequency estimation:

- averaging over n independent and identical interrogations
- fixing the total averaging time $\tau = nT$

$$\Delta\omega(T) = \frac{\Delta\phi(T)}{\sqrt{T\tau}}$$

Ideal scenario:

- disregarding decoherence: $\Delta\phi$ is independent of time and thus $\Delta\omega(T) \sim 1/T$
- standard quantum limit (SQL) $(\Delta\omega_{SQL}(T))^2 = 1/\tau T N$ for uncorrelated states
- Heisenberg limit (HL) $(\Delta\omega_{HL}(T))^2 = 1/\tau T N^2$ saturated by the 'parity-GHZ' protocol: GHZ state $|\mathrm{GHZ}\rangle = \left(|\uparrow\rangle^{\otimes N} + |\downarrow\rangle^{\otimes N}\right)/\sqrt{2}$ and parity measurement $\Pi = (-1)^N \sigma_x^{\otimes N}$

Decoherence:

- loss of coherence: $\Delta\phi(T)$ increases with $T \to$ optimal interrogation time $T_{\min}$ exists
- SQL considering individual dephasing and spontaneous decay with rates γ and Γ:

$$(\Delta\omega_{SQL})^2 = \frac{e\,(\Gamma+\gamma)}{N\tau}$$

at optimal interrogation time $T_{SQL}^{\min} = 1/(\Gamma+\gamma)$

- parity-GHZ protocol: GHZ state collapses N times faster $\to$ no gain over SQL

Consequence: Statement has often been generalized without further investigation in the sense that GHZ states generally do not lead to any improvement in the presence of decoherence.

References

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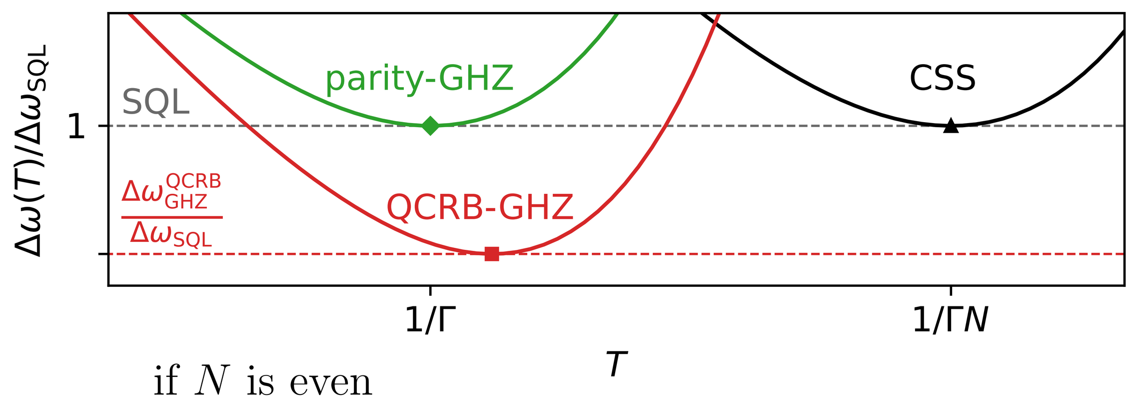
Optimal GHZ protocols

A parity measurement is not optimal in the presence of spontaneous decay since it does not saturate the quantum Cramér-Rao bound

$$(\Delta\phi_{QCRB}^{GHZ}(T))^2 = \frac{e^{(\Gamma+\gamma)NT}}{2N^2}\left[1 + e^{-\Gamma NT} + \left(1 - e^{-\Gamma T}\right)^N\right]$$

Optimal measurement:



$X = \mathcal{U}_{GHZ} S_z \mathcal{U}_{GHZ}^\dagger$ with

$$\mathcal{U}_{GHZ} = \begin{cases} \exp\left(-i\frac{\pi}{2}S_x^2\right) & \text{if } N \text{ is even} \\ \exp\left(-i\frac{\pi}{2}S_x\right)\exp\left(-i\frac{\pi}{2}S_x^2\right) & \text{if } N \text{ is odd} \end{cases}$$

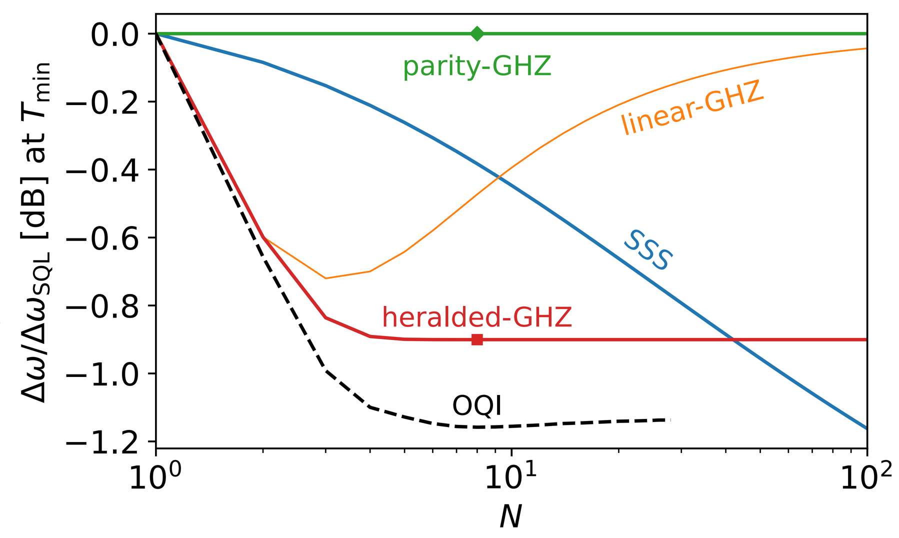
which also generates the GHZ state from the ground state $|\downarrow\rangle^{\otimes N}$ initially. Remarkably, the conditional probabilities $P(x|\phi,T)$ turn out to be independent of ϕ for all $x \neq \pm\frac{N}{2}$

$$P(x|\phi,T) = \begin{cases} \frac{1}{4}\left[1 + \left(1-e^{-\Gamma T}\right)^N + e^{-\Gamma NT} \mp 2e^{-\frac{\Gamma+\gamma}{2}NT}\cos(N\phi)\right] & \text{if } x = \pm\frac{N}{2} \\ \frac{1}{4}\binom{N}{N_-}\left[e^{\Gamma T(N-N_-)}\left(1-e^{-\Gamma T}\right)^{N_-} + e^{\Gamma T N_-}\left(1-e^{-\Gamma T}\right)^{N-N_-}\right] & \text{if } x = \frac{N}{2} - N_- \end{cases}$$

Thus, these cases don't deliver any information on ϕ.

Optimal estimation: Measurement outcomes $x \neq \pm\frac{N}{2}$ can occur only if at least one spontaneous emission event happened, such that selecting the $x = \pm\frac{N}{2}$ events in essence filters out all such cases

$$\phi_{est}(x,T) = \begin{cases} \pm e^{\frac{(\Gamma+\gamma)}{2}NT} & \text{if } x = \pm\frac{N}{2} \\ 0 & \text{else} \end{cases}$$



Together, these two features of the 'heralded-GHZ' protocol implement an error detection and mitigation scheme designated to frequency metrology.

Conclusions:

- The heralded-GHZ protocol shows a significant enhancement over SQL, independent of N for $N \geq 4$. Hence, no loss in improvement is observed for large N.
- The importance of the nonlinear estimator becomes evident since otherwise ('linear-GHZ' protocol), the QCRB is not saturated and the sensitivity converges to the SQL.
- For small ensembles, the heralded-GHZ protocol achieves sensitivities relatively close to the optimal quantum interferometer (OQI), despite its low complexity.
- For larger ensembles ($N \geq 42$), SSS achieve a higher sensitivity than the heralded-GHZ protocol and ultimately approximate the lower bound asymptotically.

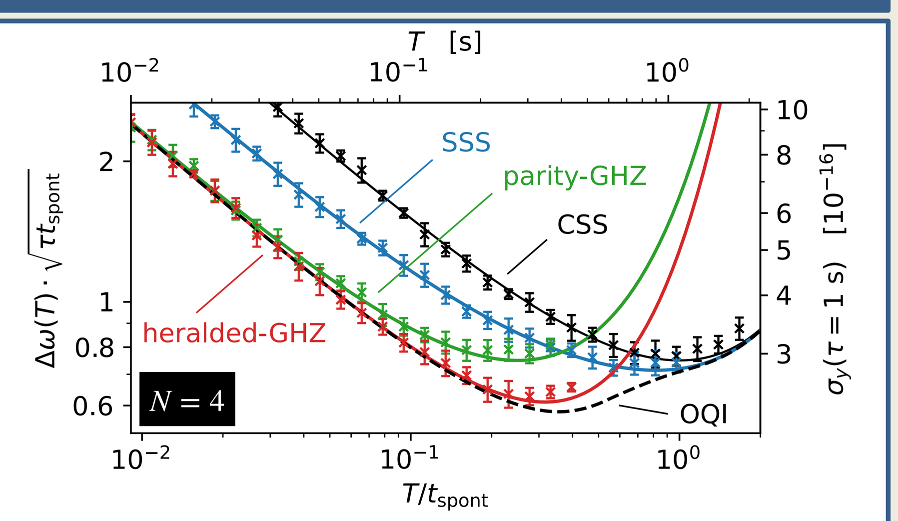
Performance in atomic clocks

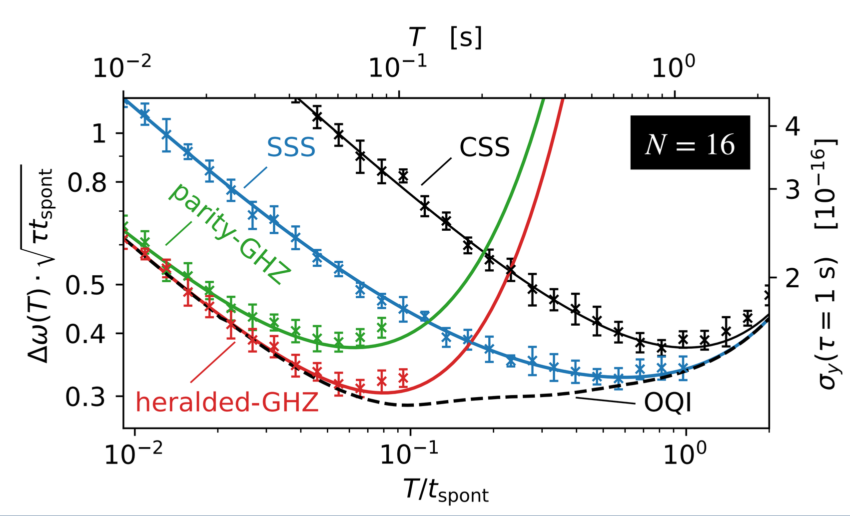
Allan deviation $\sigma_y(\tau)$ quantifies the long term stability of an atomic clock by characterizing the fluctuations of fractional frequency deviations $y = \omega/\omega_0$ averaged over $\tau \gg T$:

$$\sigma_y(\tau) = \frac{1}{\omega_0}\frac{\Delta\phi(T)}{\sqrt{T\tau}} = \frac{\Delta\omega(T)}{\omega_0}$$

Monte-Carlo simulation:

- Left/bottom: general frequency estimation, independent of τ and decay Γ.
- Right/top: Ca^+-ions clock with lifetime $t_{spont} = \frac{1}{\Gamma} \simeq 1.1$ s and transition frequency $\omega_0 = 2\pi\nu_0 \simeq 411.042$ THz.
- Frequency fluctuations: flicker noise limited state of the art clock laser with coherence time $Z \simeq 7.5$ s $\gg t_{spont}$.





Summary

- We presented a protocol with low complexity that saturates the QCRB of GHZ states.
- Significant enhancement compared to SQL in the presence of spontaneous decay.
- Measurement and estimation allow to identify and exclude spontaneous emission events $\to$ implements an error detection and mitigation scheme for frequency metrology.
- Robustness was shown through Comprehensive Monte-Carlo of atomic clocks, thereby paving the way for near-term implementations into experiments.

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