Creating Quantum Anomalies by Tightly Confining Ultracold Atoms to One Dimension

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Summary:

This poster presents models for scale-invariant, effective three- and four-body interactions of ultracold atoms confined to one dimension. The models exhibit quantum anomalies that break the scale invariance at a renormalization scale that is directly related to and controlled by the trapping potential. We describe how the direct connection between the tunable, trapping potential and the quantum symmetry breaking scale provides an avenue for probing the physics of quantum anomalies in systems of tightly confined ultracold atoms. We also discuss the direct connection in these models between the emergence of a quantum anomaly and the emergence of a topological defect, the latter of which could be exploited for generating a system with anyonic exchange statistics. Finally, we describe ideas and challenges for implementations of these models.

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[2] J. M. Leinaas and J. Myrheim, Il Nuovo Cimento B 37, 1 (1977).
[3] Y.-S. Wu, Phys. Rev. Lett. 52, 2103 (1984).
[4] N. L. Harshman and A. C. Knapp, Ann. Phys. 412, 168003 (2020).
[5] R. Jackiw, in M.A.B. Beg Memorial Volume, eds. A. Ali and P. Hoodbhoy (World Scientific, Singapore, 1991).

4 bosons in 1 1D harmonic trap, with non-local $V_{2\text{-}2}$ interaction

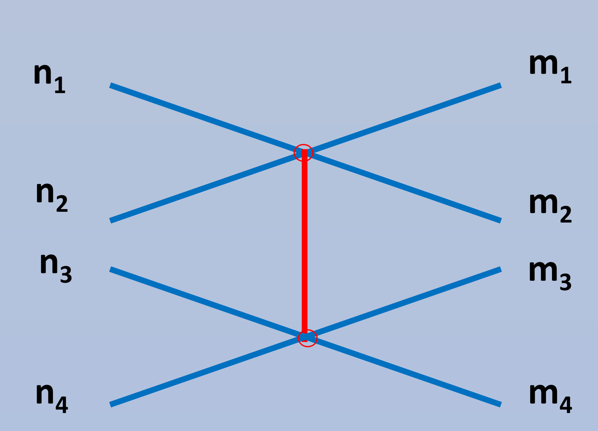
$$\hat{H}_{int} = \frac{g}{4!}\int \psi_1^\dagger\psi_2^\dagger\psi_3^\dagger\psi_4^\dagger V_{2\text{-}2}\psi_1\psi_2\psi_3\psi_4 dx_1 dx_2 dx_3 dx_4$$

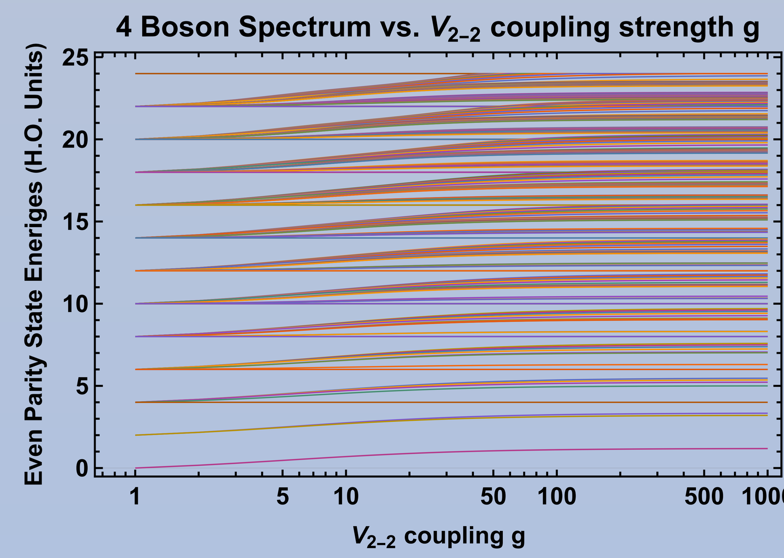
$$V_{2\text{-}2} = g(\delta_{12}\delta_{34} + \delta_{13}\delta_{24} + \delta_{14}\delta_{23}) \qquad \delta_{ij} = \delta(x_i - x_j)$$

$$J_{\mathbf{n},\mathbf{m}} = \int \phi_{\mathbf{n}}^*(\mathbf{x})\,(\delta_{12}\delta_{34} + \delta_{13}\delta_{24} + \delta_{14}\delta_{23})\,\phi_{\mathbf{m}}(\mathbf{x})\,d\mathbf{x}$$
$$= K_{n_1n_2m_1m_2}K_{n_3n_4m_3m_4} + K_{n_1n_3m_1m_3}K_{n_2n_4m_2m_4} + K_{n_1n_4m_1m_4}K_{n_2n_3m_2m_3}$$

$$K_{n_1n_2m_1m_2} = \int \phi_{n_1}^*(x)\,\phi_{n_2}^*(x)\,\phi_{m_1}(x)\,\phi_{m_2}(x)\,dx$$

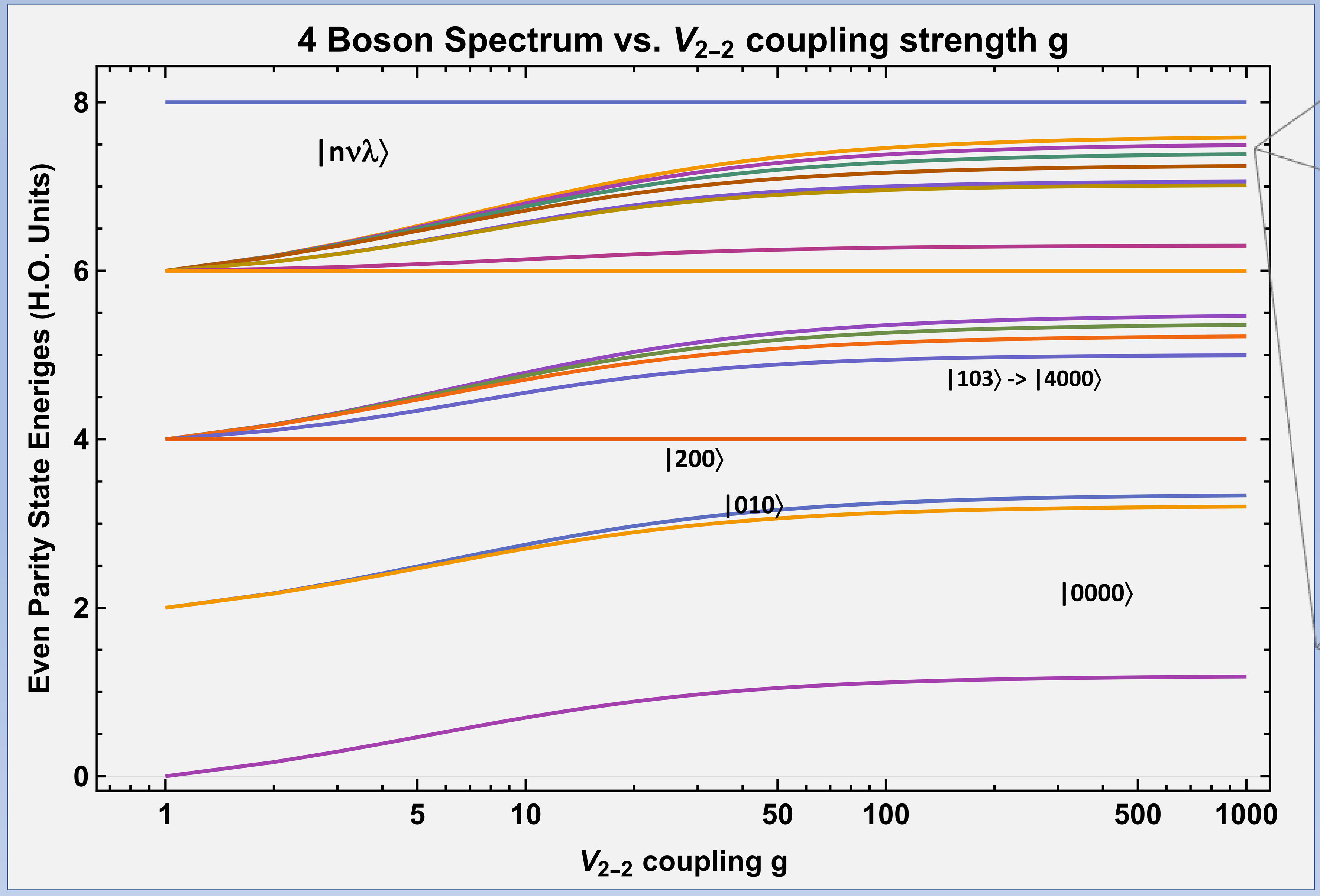
$\langle m_1m_2m_3m_4|V_{2\text{-}2}|n_1n_2n_3n_4\rangle$



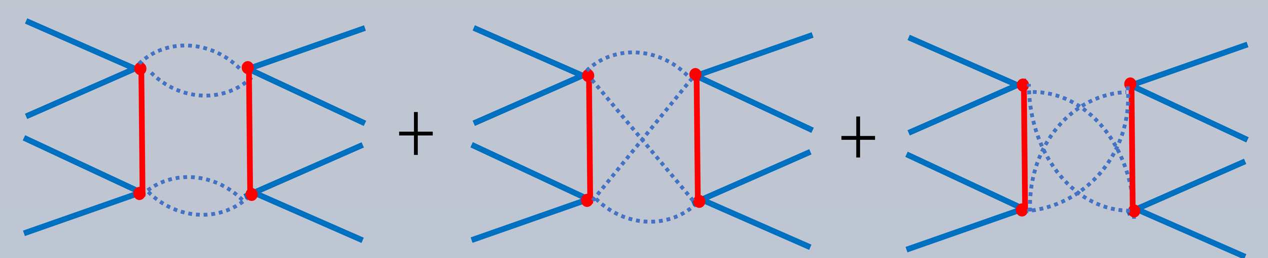


E = (n + 2ν+λ)

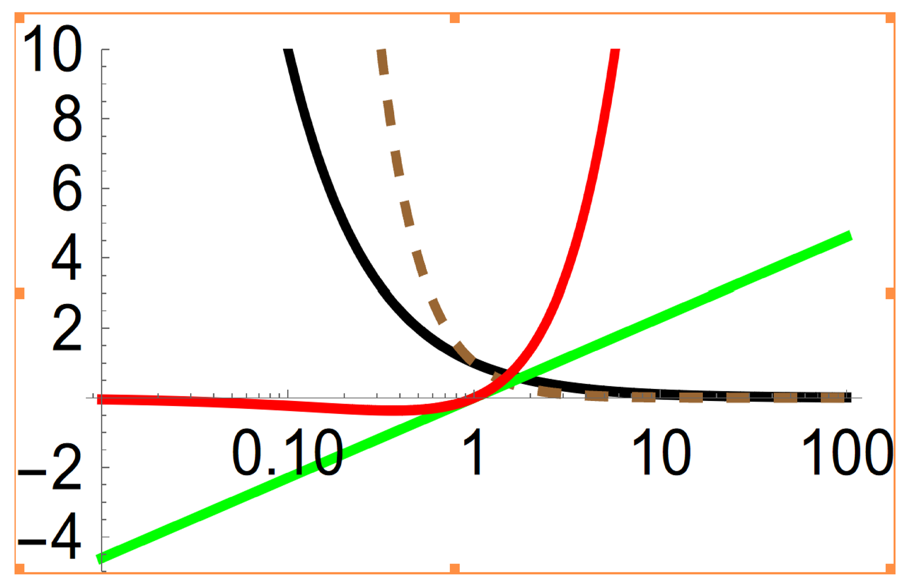
$$|\mathbf{n}\rangle = \mathcal{N}_{\mathbf{n}} \sum_{\pi\in S_4} |n_{\pi(1)}n_{\pi(2)}n_{\pi(3)}n_{\pi(4)}\rangle$$



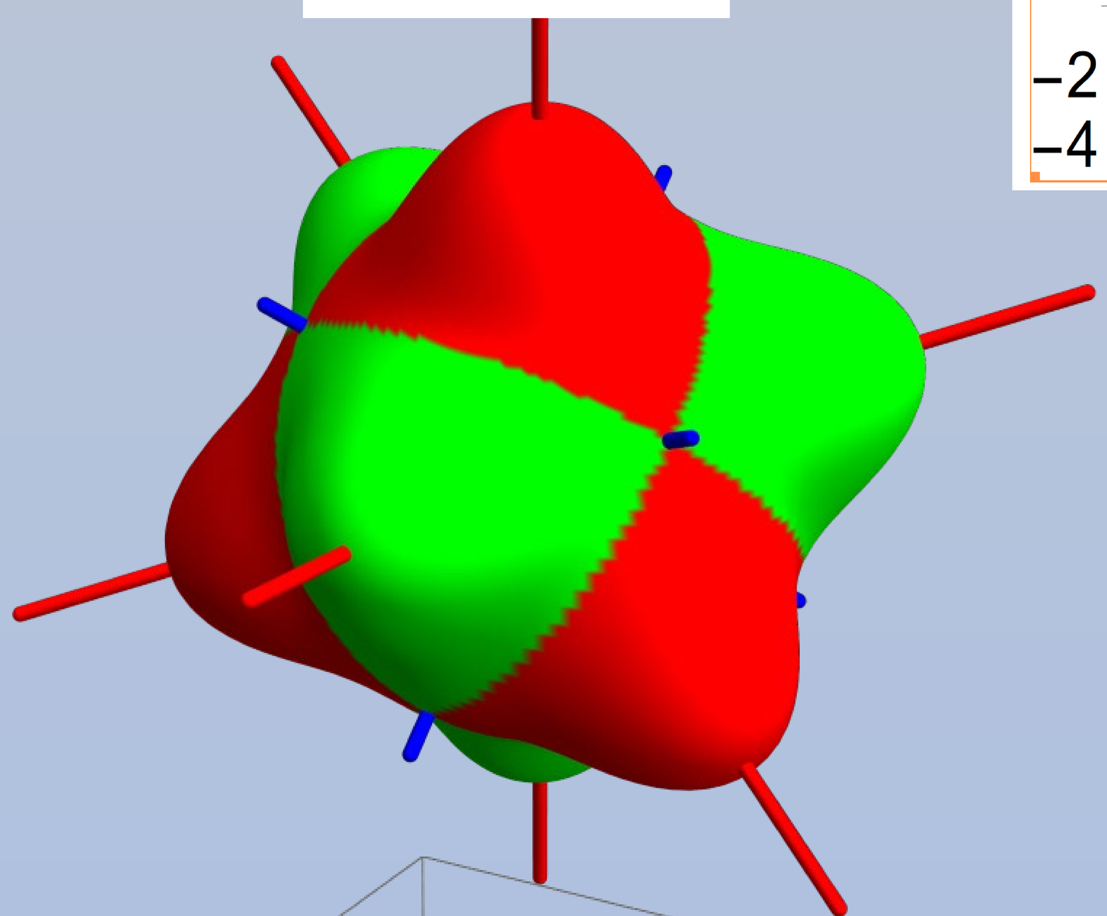
Second order correction to ground state energy, expected log divergence



$$E(L) = \delta E^{(1)}(L) + \delta E^{(2)}(L)$$
$$= \frac{g_r}{L^2} + 4!\left(\frac{4\mu}{\pi\hbar^2}\right)\frac{g_r^2}{L^2}\log(L/L_r) + \mathcal{O}(g_r^3) + \mathcal{O}(1/\Lambda)$$



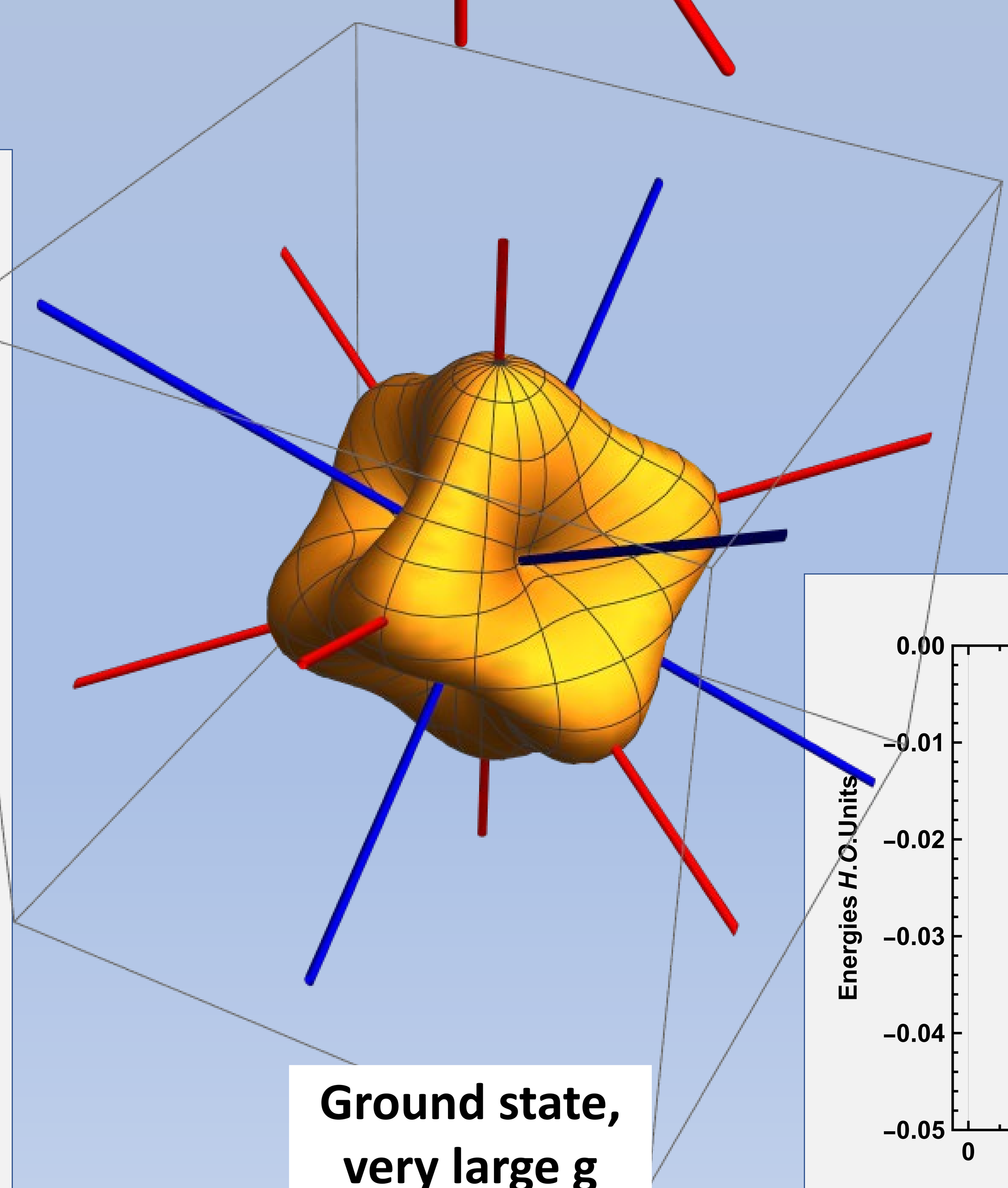
First Dark state



Wavefunctions in Jacobi coordinates

$$\cos\theta = z_3/\rho,\quad \tan\phi = z_2/z_1,\quad \rho^2 = z_1^2 + z_2^2 + z_3^2$$

$$z_0 = (x_1 + x_2 + x_3 + x_4)/2$$
$$z_1 = (x_1 - x_2)/\sqrt{2}$$
$$z_2 = (x_1 + x_2 - 2x_3)/\sqrt{6}$$
$$z_3 = (x_1 + x_2 + x_3 - 3x_4)/\sqrt{12}$$



Ground state, very large g