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The Interplay between Health Conditions and the Annuity Puzzle

Abstract: Planning for retirement involves complex choices that often challenge the assumptions of rational decision-making. The Annuity Puzzle is a phenomenon associated with the individual preference for non-lifetime incomes despite theoretical results indicating that choosing lifetime annuities maximizes utility. This research focuses on the Annuity Puzzle, integrating health conditions with actuarial and economic analyses. The objective is to contribute to understanding retirement decision-making, particularly in the Supplemental Pension System (*RPC*) context. A simulation model based on the Gompertz distribution was developed to estimate the Money's Worth Ratio (*MWR*), which compares the present value of lifetime annuity payments to the cost of purchasing the annuity. The aim was to estimate possible deviations from an actuarially fair annuity. The simulations employed current Brazilian mortality data to evaluate how different mortality profiles related to primary fatal health conditions might be relevant based on microdata related to Brazilian mortality in 2022. The results highlight the need to consider the impact of health on retirement planning and annuitization decisions in Brazil. The *MWR* results obtained for the major underlying causes reveal that even actuarially fair annuities are often unattractive to those with lower life expectancy due to chronic diseases, with an average corresponding to two-thirds for the most severe case. Discrepancies between life expectancy in annuity pricing and actual mortality data can lead to adverse selection. However, aligning health and retirement strategies can establish a more equitable retirement framework in Brazil, aiming a greater financial security.

Keywords: Annuity Puzzle, Life Annuities, Health Conditions, Money’s Worth Ratio, Supplementary Pensions.

1. Introduction

Retirement planning, a labyrinth of intricate and often irrevocable choices, is riddled with challenges that deviate from rational economic principles. One of the most intriguing anomalies in this realm is the Annuity Puzzle, a concept first brought to light by Modigliani (1986) based on the seminal work of Yaari (1965), a historical milestone in the field. This puzzle underscores a trend by which individuals frequently opt for non-lifetime retirement incomes over annuities, which defies economic theories advocating for annuities to maximize expected utility over an uncertain lifespan.

The Brazilian retirement landscape, as delineated by the Ministério da Previdência Social (2023), serves as a backdrop for investigating the Annuity Puzzle. The population covered by the Supplementary Pension Regime (Regime de Previdência Complementar or RPC) grew by approximately 5.6% between 2014 and 2023, reaching 14.7 million people—about 7% of the Brazilian population. Open plans, offered and managed by Open Pension Entities (Entidades Abertas de Previdência Complementar or EAPCs), previously catering to 10.7 million people, witnessed a growth of 3.7%, expanding to 11.1 million. In turn, closed plans, managed by closed pension entities (Entidades Fechadas de Previdência Complementar or EFPCs) and having more withdrawal restrictions, experienced a growth of about 9.3%, reaching 3.6 million people.

These figures, although significant, hide a great expansion, as the total population reached 16.7 million people at its peak in 2020. The COVID-19 pandemic likely impacted retirement reserves due to liquidity concerns, leading to a 12% decrease in the population covered by the *RPC*. Open plans, which allow withdrawals more freely, decreased by 14,6%, over three times more than the decrease in closed plans (minus 4,6%). The financial growth of these entities, however, is still particularly notable.

Open plans saw a 211% increase in assets, from R$470 billion to around R$1.46 trillion. Meanwhile, closed plans experienced an 83% growth, from R$700 billion to around R$1.28 trillion. As for the benefits paid, in 2023, the benefits paid amounted to R$94 billion, representing 1.06% of the Gross Domestic Product (GDP), with 95% disbursed by the *EFPCs*. A noticeable aspect of the *RPC* is that most participants are still in the accumulation phase, highlighting the potential for future growth. This is evident compared to the R$372.4 billion in benefits issued by the General Social Security Regime (Regime Geral de Previdência Social or RGPS) in 2022 (Ministério da Previdência Social, 2023b).

Yaari (1965) seminal work on annuities posits that lifetime annuities yield superior returns due to the Mortality Premium. In this concept, individuals who live longer benefit from the premiums of those who pass away earlier. However, Yaari's model simplifies the assumption that individuals of the same age have equivalent life expectancies, disregarding variations in biological age, health conditions, and associated health costs. This assumption underscores the need for a comprehensive understanding of retirement decisions that incorporate health conditions—a gap this study is dedicated to contributing to.

To address the gap in the literature concerning health statuses, the present study integrates the complexities of the Brazilian *RPC* with empirical findings on the Annuity Puzzle and the mortality trends observed in the Mortality Information System (Sistema de Informação sobre Mortalidade or SIM). By leveraging actuarial simulations and the Money's Worth Ratio (*MWR*) as a critical indicator, this study evaluates the actuarial fairness of lifetime annuities offered within the *RPC*. The *MWR* is a measure that compares the present value of the expected annuity payments to the cost of the annuity, providing an indicator of the value of the annuity.

Integrating public health and actuarial science concepts allows for clustering different health conditions, offering insights into how these conditions impact the *MWR* of annuities. This approach bridges theoretical economic models with empirical aspects of retirement in Brazil, informing tailored financial planning strategies that account for individual health statuses alongside economic objectives.

This research aims to enhance the understanding of retirement decisions, providing insights for policy development and individual financial planning. By addressing the interplay between health conditions and annuitization within the context of the Brazilian *RPC*, this study contributes to a more comprehensive understanding of the Annuity Puzzle and its implications for retirement planning.

This paper is divided into five sections, including this brief introduction. The second section presents the theoretical framework for supplementary pensions and some evidence on the Annuity Puzzle. It also presents the main specificities of the *RPC* in Brazil. The third section describes the methodology used. The fourth section reports the results obtained. The last section presents the final considerations.

2. Theoretical Basis and Empirical Literature

2.1 Theoretical Basis for the Annuity Puzzle

Yaari (1965) posited that under certain conditions, lifetime annuities are the optimal choice for individuals wishing to maximize their expected utility and consumption throughout their uncertain future lifetime, starting from retirement. He argued that lifetime annuities outperform traditional investments due to the "Mortality Premium," wherein longer-living individuals are subsidized by those who die prematurely. Modigliani (1986) supported these conclusions, emphasizing that lifetime annuities provide a regular income stream, mitigating the risk of running out of funds while still alive.

Benartzi et al. (2011) agreed that lifetime annuities minimize the risk of depleting resources, sparing individuals the challenge of determining an optimal drawdown rate and offering more predictability in budgeting. This reassures individuals about their financial decisions and the role of lifetime annuities in maximizing their expected utility. In summary, lifetime annuities offer increased consumption and risk mitigation. However, Modigliani (1986) highlighted the "Annuity Puzzle," a term that describes the discrepancy between the theoretical benefits of lifetime annuities and their actual usage.

The empirical uptake of lifetime annuities is far lower than theoretical models predict. Rusconi (2008) illustrated this by noting that in a study of annuity markets, only twenty-seven countries were mentioned, with substantial markets in only the UK, Switzerland, Denmark, and South Africa. Brazil was not mentioned, denoting the relative modesty of the Brazilian *RPC*, which as shown in the introduction, barely covers 10 percent of the Brazilian population. This underscores the importance of public policy in addressing this issue, particularly in light of the income and expenditure profile of most of the population, which is inflicted by a notorious social inequality, as depicted by a GINI Index of 52.0 in 2022 (World Bank Group, 2024).

Yaari's model assumed a representative individual maximizing intertemporal utility in an environment where the only uncertainty is lifespan, without any bequest motives, and with "actuarially fair interest rates," which can be unfeasible in most cases. Aiming to address that, in a less restrictive model, Davidoff et al. (2005) concluded that actuarially fair annuities or utilities following Von Neumann and Morgenstern's axioms are not essential.

This finding suggests that some unfairness and deviation from a maximized utility might be acceptable. Although the actual acceptancy frontier of unfairness concerning annuities is unknown to the best of our knowledge, the findings about the ultimatum game suggest there is a limit on this matter (Güth & Van Damme, 1998; Berger et al., 2012; Arvanitis et al., 2019). Nonetheless, lifetime annuities are still optimal in complete markets, provided there are no bequest motives, and returns exceed those of equivalent-risk conventional investments.

Furthermore, Goedde-Menke et al. (2014) highlighted that a mixed portfolio, which includes lifetime annuities, can be flexible and optimized, even with bequest motives. This underscores the adaptability of annuitization, even when the individual has a bequest motive. In presenting alternatives to address the bequest motive, Sutcliffe (2015) suggested arbitrage opportunities with life insurance policies under certain conditions.

As the life-cycle models fail to fully resolve the Annuity Puzzle, suggesting it goes beyond the *homo economicus* archetype, Davidoff et al. (2005) proposed that behavioral aspects could be the key to understanding the lack of demand for lifetime annuities. This call for more behavioral approaches has led to innovative pathways, with subsequent studies incorporating a range of topics such as risk aversion (Hu & Scott, 2007), preference for leisure (Chai et al., 2011), the time preference rate (Cappelletti et al., 2013), complexity (Brown et al., 2013), myopia (Previtero, 2014), the elasticity of intertemporal substitution (Horneff et al., 2015), and mortality salience (Salisbury & Nenkov, 2016). This collaborative research has enriched our understanding of the annuity puzzle and made us feel part of a community striving to solve this complex phenomenon.

2.2 Empirical Literature

Bütler and Teppa (2007), in their analysis of Swiss pension plans, noted the surprisingly few empirical studies that try to explain why individuals do not annuitize their resources. They stressed the need for more reliable data, an essential aspect of our research. Alexandrova and Gatzert (2019), in their literature review of the 89 Annuity Puzzle-related articles, corroborate this perception by finding only seventeen to be predominantly empirical. These studies often focused on mandatory supplementary plans within Pillar 2 (Holzmann et al., 2008), as is the case of the already mentioned Bütler and Teppa study.

Despite the challenges posed by data limitations, many authors have turned to theoretical model-based simulations. Their findings are intriguing. For instance, Dus et al. (2005) discovered that delaying annuitization could optimize benefits and minimize resource exhaustion risks in Germany. Horneff et al. (2008) concluded that a moderately risk-averse individual in the US would likely annuitize 60% of their assets. On the other hand, De Villiers-Strydom and Krige (2014) found non-annuitization to yield the best results in South Africa. Conversely, Chalmers and Reuter (2012) and Previtero (2014) noted that recent stock market returns, at least in the US, increased the likelihood of lump-sum payments, which was influenced by how actuarially fair the annuity was when compared with potential stock returns.

Behavioral biases, context, and personal preferences have also been examined. For instance, Poterba et al. (2011) highlighted the tendency for individuals to default to the plan's default option.

Similarly, Shu et al. (2016) emphasized the crucial role of the framing effect in the presentation of lifetime annuities, underscoring the significant impact of the framing effect on decision-making. In another avenue, Schreiber and Weber (2016) noted intertemporal choice inconsistency, where individuals might prefer lump-sum payments despite initially favoring lifetime income due to hyperbolic discounting.

2.3 Our Approach to the Annuity Puzzle

Brazilian literature sparsely addressed the benefit perception phase and its chosen process within the *RPC*, focusing instead on legal impacts, sectoral aspects, and fund performance (Campani et al., 2020). In contrast, as analyzed in the work by Alexandrova and Gatzert (2019), international literature has extensively explored retirement decisions, with numerous studies highlighting the existence of the Annuity Puzzle. However, according to that work, among 89 articles that address this conundrum, only one explores health considerations from a theoretical model (Reichling & Smetters, 2015), highlighting the need for more theoretical models to study health considerations.

Three other articles have delved into health coverage datasets related to national health systems or occupation pension plans, providing empirical evidence on annuitization rates in different countries (Brown, 2001; Hagen, 2015; Koijen et al., 2016). Nonetheless, these studies present a bias throughout mandatory supplementary plans within the scope of Pillar 2, which are mandated occupational or personal pension schemes often regulated to promote annuitization (Holzmann et al., 2008). The Brazilian *RPC*, also occupational or personal schemes outside social public schemes, is based on voluntary opt-in and opt-out, thus within the Pillar 3 framework, which is the focus of this study.

Our approach focuses on three key factors related to the Annuity Puzzle and health statutes within optional supplementary plans. The first factor analyses how poor health leads to adverse selection. This topic is covered in fifteen articles revised by Alexandrova and Gatzert (2019): Brown (2001), Valdez et al. (2006), Gupta and Li (2007), Milevsky and Young (2007), Horneff et al. (2008a), Horneff et al. (2008b), Webb (2009), Chalmers and Reuter, (2012), Wang and Young (2012a), (Cappelletti et al., 2013), Pashchenko (2013), Kling et al. (2014), Liang et al. (2014), Guillemette et al. (2015), and Hagen (2015). As we conclude, our goal is to comprehensively understand this topic in the Brazilian setting.

The second subject is loss of liquidity, especially when medical expenses and health shocks present a bankruptcy risk due to circumstantial conditions, as typically observed in the United States, where most studies were conducted. Fourteen articles listed by Alexandrova and Gatzert (2019) mentioned that possibility: Davidoff et al. (2005), Hu and Scott (2007), Horneff et al. (2008a), Pang and Warshawsky (2010), Ameriks et al. (2011), Hwang et al. (2012), Lockwood (2012), Chalmers and Reuter, (2012), Wang and Young (2012a), Wang and Young (2012b), Pashchenko (2013), Reichling and Smetters (2015), Peijnenburg et al. (2016) and Ai et al. (2017). Due to the Unified Health System (Sistema Único de Saúde or SUS), the loss of liquidity due to health concerns may not be as pronounced in Brazil as in the United States. However, it is still an important aspect to be considered, as not all types of expenses are covered by the SUS, and the coverage provided may not be timely.

The third concerns one's perception of one's life expectancy. Alexandrova and Gatzert (2019) present six articles that approach this topic: Brown (2001), Inkmann et al. (2011), Beshears et al. (2014), Goedde-Menke et al. (2014), Banks et al. (2015), Hagen (2015), and Wu et al. (2015). This topic is closely related to poor health prospects and is therefore of paramount importance. However, self-perception's subjective nature presents significant challenges, making this topic intriguing and engaging.

It is important to note that few studies on the Annuity Puzzle have been published since 2020, likely due to the significant attention and patronage of COVID-19-related research in recent years. To name a few, those authors have contributed to this field in the last four years: d'Albis et al. (2020), Lambregts and Schut (2020), Asher, (2021), Han and Hung (2021), Mindlin (2021), Boyle et al. (2022), Chen and Rach (2022), Jong and Robinson (2022), Pashchenko and Porapakkarm (2022), Zhang et al.

(2021), De Villiers-Strijdom and Krige (2023), Jeong et al. (2023), Korankye et al. (2023), Look et al. (2023), MacMinn and Ren (2023), O'Dea and Sturrock (2023), and Pashchenko and Porapakkarm (2024). This study continues the recent trend in the field, by which more attention is given to differences in mortality rates among the population.

2.4 The Brazilian Supplementary Pension System (*RPC*)

Benartzi et al. (2011) observed that the Annuity Puzzle is rooted in personal preferences, cognitive biases, and the structure of annuity offerings. Therefore, it is essential to consider some Brazilian specificities in which annuities are offered. As part of Pillar 3 (Holzmann et al., 2008), the Brazilian *RPC* is governed by Complementary Law No. 109/2001. It comprises entities managing voluntary, complementary, and autonomous benefit plans. The *RPC* has open and closed segments.

Open Segment: This segment is dedicated to *EAPCs*, regulated by the National Council of Private Insurance (Conselho Nacional de Seguros Privados or CNSP) and supervised by the Private Insurance Superintendence (Superintendência de Seguros Privados or SUSEP). *EAPCs* include for-profit joint-stock companies and insurance companies offering survival coverage plans, such as the Life Insurance Free Benefit (Vida Gerador de Benefícios Livres or VGBL) and Life Insurance Immediate Income (Vida de Renda Imediata or VRI). These plans are like the Free Benefit Scheme (Plano Gerador de Benefícios Livres or PGBL) and Immediate Income Scheme (Plano de Renda Imediata or PRI) but differ legally, with contributions considered insurance premiums and non-deductible from personal income tax.

Closed Segment: Comprises *EFPCs*, regulated by the Supplementary Pension Management Council (Conselho de Gestão da Previdência Complementar or CGPC) and supervised by the National Superintendence of Supplementary Pensions (Superintendência Nacional de Previdência Complementar or Previc). *EFPCs* are non-profit civil societies or foundations, and scheme participation is restricted to individuals with employment, associative, or corporate ties to the sponsor (who contributes to the plan) or institutor (who does not). Sponsors can be private or public entities, with additional regulations under Complementary Law No. 108/2001 for public entities.

Benefit schemes are classified based on the contribution-benefit relationship into Defined-Benefit (Benefício Definido or DB), Defined-Contribution (Contribuição Definida or DC), and Variable-Contribution (Contribuição Definida or VC) modes:

- ***DB* Schemes:** Offer benefits based on a calculation rule that considers salary history or a strict contribution setting. The contributions might be actuarially adjusted to ensure the benefit's value.
- ***DC* Schemes:** The benefit value adjusts according to the participant's account balance during the accumulation and benefit phases.
- ***VC* Schemes:** These schemes combine *DB* and *DC*. This means that contributions are treated as a *DC* Scheme. In contrast, benefits are treated as a *DB* Scheme, either lifelong (thus, an annuity benefit) or non-lifelong benefit (thus a non-annuity benefit). It is essential to recall that Barr & Diamond (2006) define both *DC* and *VC* schemes under pure *DC* or non-pure *DC* Schemes, where benefits depend on accumulated resources. Pure *DC* Schemes lack lifelong benefits, leaving longevity risk entirely with the individual, which can have significant implications.

With those two definitions groups, the following association can be made concerning the normative characteristics of open and closed entities:

EAPC Schemes: Per the *CNSP* Resolution No. 349/2017, pension schemes can be formally classified as *DB* or *VC*. *DB* schemes have pre-defined benefits and contributions. *VC* schemes, on the other hand, base benefits on the accumulated balance of the Mathematical Provision of Benefits to be Granted (Provisão Matemática de Benefícios a Conceder or MPBG). It is crucial to understand that the *VC* definition is specific to the Brazilian setting and is different for open and closed entities. For open entities, the benefit will be considered *VC* as long the benefit becomes well known once the retiree opts for a benefit option, regardless of whether it is lifelong. Thus, it is essential to remember the guidance of Barr & Diamond (2006), who defined both *DC* and *VC* schemes under Defined-Contribution Schemes, where benefits depend on accumulated resources. Pure *DC* Schemes lack lifelong benefits, leaving longevity risk entirely to the individual, which can have significant implications. On the other hand, non-pure *DC* Schemes, or *VC* schemes according to Brazilian regulations, may offer lifelong benefits.

EFPC Schemes: Per *CGPC* Resolution No. 16/2005, these can be classified as *DB*, *DC*, or *VC*. *DB* schemes, which are tied to the salary history parameter, are not the focus of this study. *VC* schemes operate as previously described, with the lifelong benefit option being necessary. However, unique to *EFPCs*, *DC* Schemes stand out by offering only non-lifelong benefits with variable or indefinite terms, thereby exposing individuals to the significant risk of outliving their retirement savings.

For those approaching retirement, it is crucial to understand the potential risks associated with lump-sum options. These options can lead to rapid resource depletion and come with tax penalties, even in *VGBL* schemes where the interest earned is subject to taxation. Non-lifelong benefit options offered by closed entities may come with fluctuating incomes and uncertain benefit durations due to the risk of insufficient long-term returns. For open entities, the non-lifelong options follow an agreed interest rate, which revolves around a locked interest rate scenario.

The cash flow design of non-lifelong benefits varies between open and closed entities. However, any non-lifelong benefits are unrelated to the retiree's survival and death, meaning that while the remainder balance is still heritable, the longevity risk is unaddressed. Given the outlined information provided concerning the *RPC*, as of December 2023, 509 *DC* schemes were managed by *EFPCs*, 88 (20.9%) more than the 421 observed in 2014. In the same period, there was a notable decrease in *VC* schemes by 26 (7.2%), from 360 to 334, and in *DB* schemes by 35 (10.7%), from 327 to 292. This decline in *DB* and *VC* schemes underscores the shift towards pure *DC* schemes within supplementary pension systems, as Holzmann et al. (2008) highlighted.

3. Methodological Procedures

3.1 Simulation Model

The model used for simulation is based on the classic Gompertz distribution (1825), a historically significant model widely adopted in survival analysis studies. This distribution allows approximating traditional biometric tables using continuous functions, which are computationally more convenient. The following expression gives the force of mortality for this distribution: α is the parameter indicating the force of mortality at birth, and β indicates the growth rate as a function of age $\boldsymbol{x}$. The coefficient values can be estimated by the Ordinary Least Squares Method (Humes et al., 1984):

$$\mu(x) = \alpha e^{\beta x};\ \alpha > 0, \beta \in [0, \infty) \tag{1}$$

Based on the relations presented by Bowers et al. (1997), Milevsky (1998), and Dickson et al. (2009), the value of a Whole Life Immediate Annuity that pays one monetary unit at the end of each year as long as the annuitant is alive is calculated using the following equation:

$$a_x = \sum_{t=0}^{\infty} v^t\, {}_t p_x \tag{2}$$

Where $\boldsymbol{a_x}$ is the actuarial notation for an Immediate Annuity; $\boldsymbol{v^t}$ is the discount factor, and ${}_t\boldsymbol{p_x}$ is the conditional probability of an annuitant of age $\boldsymbol{x}$ surviving for more $\boldsymbol{t}$ years. It's worth noting that annuities with annual payments, as represented by $\boldsymbol{a_x}$ in actuarial notation, are a rarity, as pointed out by Dickson et al. (2009). These annuities, which pay out once a year, are not the norm, as payments usually occur more frequently. However, their rarity does not diminish their importance, especially considering the limitations in mortality data between integer ages, typically based on integer age life tables.

The interest rate is a pivotal factor that significantly impacts the cost of an annuity. As Yaari (1965) points out, the instantaneous interest rate at time $\boldsymbol{t}$, denote as $\boldsymbol{r(t)}$, is expected to be higher than the market interest rate $\boldsymbol{j(t)}$. However, it is crucial to remember that the interest rate is not a fixed entity but rather contingent on the insurance company's behavior. Therefore, Yaari's assumption that the actuarial rate of interest is fair in the actuarial sense is a guiding principle. Davidoff et al. (2005) further underscore that annuities need not be actuarially fair but must offer favorable net premiums over conventional assets.

Following this, the Gross Annual Income (*GAI*) is calculated as follows:

$$GAI(a_x, W_0) = \frac{W_0}{a_x} \tag{3}$$

Where $\boldsymbol{a_x}$ is the Immediate Annuity and $\boldsymbol{W_0}$ is the amount paid for a given annuity, usually using the *MPBG* balance, which is the individual's retirement account. According to (Assaf Neto, 2006), the Present Value Interest Factor (*PVIF*) of a given annuitant can be estimated with the following equation:

$$PVIF(i, T_x) = \frac{1-(1+i)^{-T_x}}{i} \tag{4}$$

Where $\boldsymbol{i}$ is the interest rate and $\boldsymbol{T_x}$ is the future lifetime at age $\boldsymbol{x}$, which is a random variable under a probability distribution, which can be derived from the following equation:

$$F_X(x)^{-1} = \frac{\ln\left(\beta\frac{\left(\frac{\alpha}{\beta}\right)-1\ (1-U)}{\alpha}\right)}{\beta};\ U \sim u(0,1) \tag{5}$$

Where $\boldsymbol{F_X(x)^{-1}}$ is the quantile function or the inverse of the cumulative distribution function of the Gompertz Distribution (Gompertz, 1825) and $\boldsymbol{U}$ is a random number derived from a Uniform Distribution in the interval (0,1). By applying this expression recursively $\boldsymbol{n}$ times, a synthetical population of individuals $\boldsymbol{m}$ are created.

Returning to equation (3), the Net Present Value (*NPV*) of an annuitant is calculated as:

$$NPV(GAI, PVIF) = GAI \times PVIF \tag{6}$$

Leading to the Money's Worth Ratio (*MWR*) as shown in equation (7):

$$MWR(W_0, NPV) = \frac{NPV}{W_0} \tag{7}$$

The interpretation of the Money's Worth Ratio is as follows: if the result equals one, the individual will receive at the end what they had paid for at the beginning. From the population perspective, the same result means that the annuity is as actuarially fair as possible.

3.2 Assumptions

An annuity is actuarially fair when the expected cash flow of the benefits meets the amount paid for at the beginning of the de-accumulating phase. Therefore, for theoretical purposes, its calculation is supposed to hold a loading rate $\boldsymbol{L}$ equal to zero (Milevsky, 1998), with a force of mortality rate consistent with the population (Milevsky & Young, 2007) and a real interest rate compatible with the long-term average returns (Milevsky, 2001).

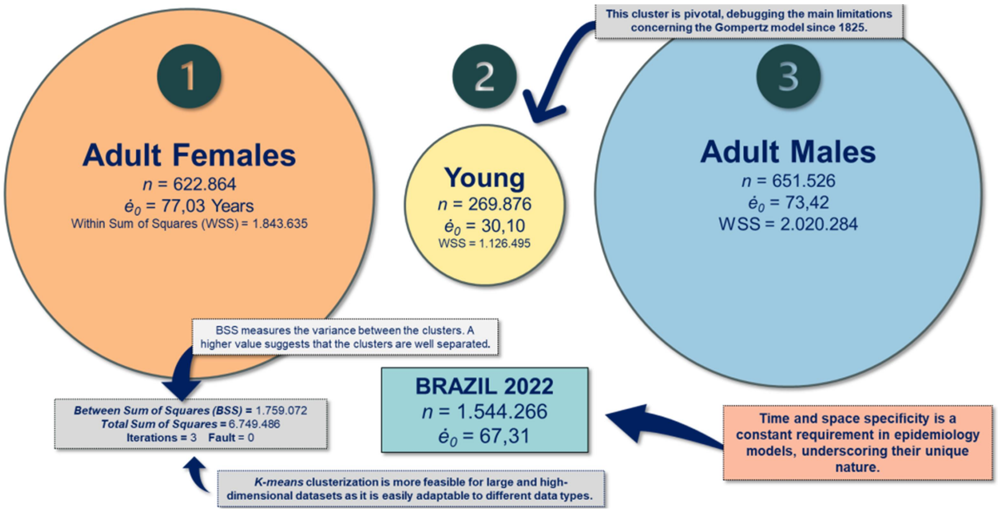
The mortality assumptions forming the actuarially fair scenario follow the mortality rate based on the mortality table IBGE 2022 extrapolated by the Ministry of Social Security (Ministério da Previdência Social or MPS), segregated by sex and translated into Gompertz distribution parameters. This rate comprises the Brazilian population's life expectancy while preserving the skewness and kurtosis profile across smoothed or aggravated variations.

We have chosen 3% per year for the long-term real interest rate to calculate the *PVIF*, for the actuarially fair scenario, based on a perspective of a decline in the neutral interest rate, supported by strengthening the Brazilian economy's fundamentals in the long run. Although this assumption is discretionary, this interest level prevents distortions derived from short-term Brazilian interest rate trends, which are sensitive to conjectural impacts.

3.3 Cluster Analysis

To assess specific mortality trends among the most significant causes of death, a K-means clusterization procedure was conducted on Brazilian mortality data in 2022, using the data provided by the Mortality Information System (SIM) to estimate the mean age of death within each cluster, providing the actual life expectancy within each group, without lending distortions derived from other groups. The main population clusters are depicted in Figure 1, as follows:

Figure 1 – Statistics of the Main K-means Clusters in 2022 Brazilian Mortality Data



Unlike hierarchical clusterization, where clusters are selected upon a visual dendrogram selection, K-means clustering requires a predefined number of clusters. The choice of 3 clusters aimed to isolate the impact of infant, youth, and early adulthood mortality, which distorts Gompertzian fitting adherence, keeping adult gender-specific clusters for further analysis. The two-parameter Gompertz model is conceived for natural causes of death through non-neonate lifespan stages, and this data wrangling procedure upon a micro-level dataset aided the depuration of the dataset for adulthood onward mortality analysis. This step was crucial, as actual mortality rates for the younger population in Brazil are significant.

Infant mortality, for instance, presents a force of mortality at birth ($\boldsymbol{q_0}$) over 35 times greater than the one estimated on the BR-EMSsb-v.2021 life table ($\boldsymbol{q_0}$ = 0.010321 against $\boldsymbol{q_0}$= 0.000293, for females). The BR-EMSsb-v.2021, along with the AT2000 life table, will be used in our annuity calculations to denote the deviation between the prospective life expectancy of a given group of individuals who will eventually pass away due to a given underlying cause of death.

Therefore, the IBGE 2022 Extrapolated by the MPS, with the adaptability of the $\boldsymbol{q_x}$ life table variable, can be easily adjusted by aggravating (A %) or smoothing (S %) the force of mortality. Table 1 illustrates the leading underlying causes of death by their corresponding 3-character *ICD-10* (International Classification of Diseases tenth version) code for the female cluster, which will reference the results provided in the fourth section. The results in Table 1 correspond to key percentile ages of death observed for each underlying cause within the mortality dataset and their corresponding smoothed or aggravated Gompertzian life expectancy. The 99 percentile was used as *SIM* data, initially truncated to the age of 99, and Gompertzian estimates may be misleading at supercentenarian ages (Tolley et al., 2016).

Table 1 – Main Underlying Causes of Death and Their Life Expectancies

Underlying Cause Group	Min. Age	Mean Age	Smooth/ Aggrav.	Gompertz Expectancy	P99 Age
Malignant neoplasms	35.1321	69.93736	A127	69.94114	96.46366
Diabetes mellitus and endocrine disorders	35.83025	75.87615	A41	75.91868	100.23463
Cardiovascular diseases	35.69336	77.54579	A21	77.75052	101.35524
Infectious and parasitic diseases	40.18617	77.97962	A19	77.94806	101.15266
Respiratory infections	40.06297	82.22676	S18	82.28158	103.18322
Neuropsychiatric conditions	35.24709	82.74788	S21	82.70878	101.97027

4. Key Results

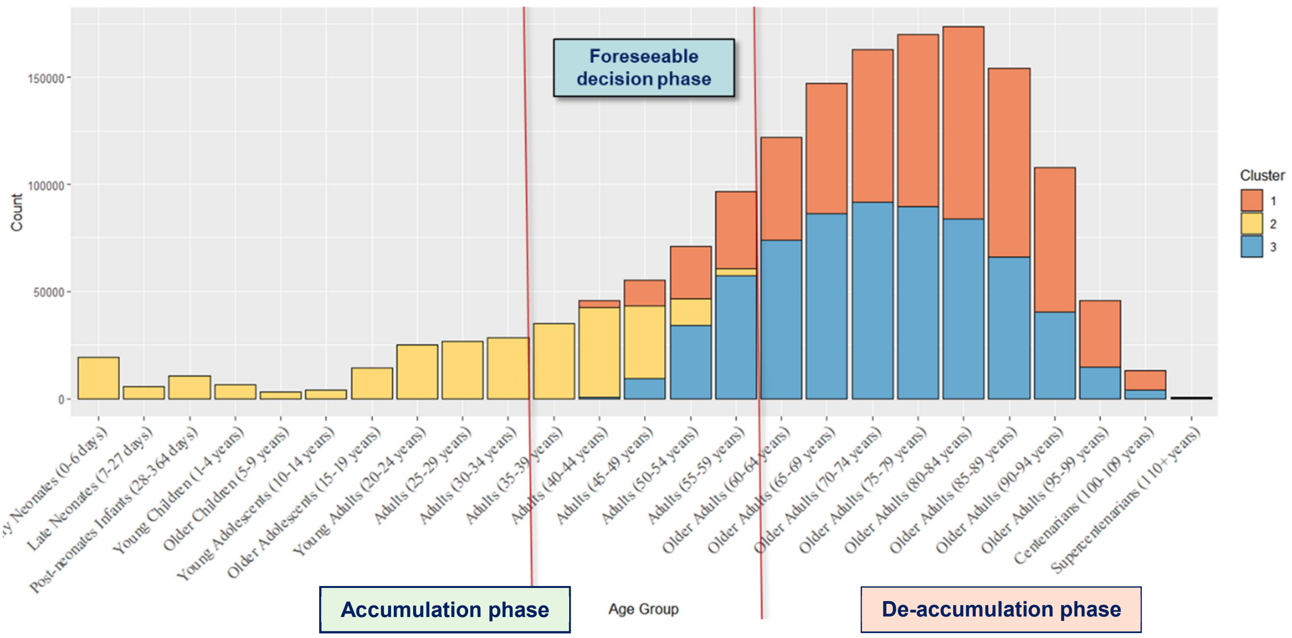
4.1 Population Mortality Profile

Supplementing the insights from Figure 1, Figure 2 illustrates the mortality distribution by age group. The data plays a crucial role in shaping retirement planning, as it shows that at a retirement age of sixty, no individuals from the younger age group in cluster two are alive during the de-accumulation phase. This indicates that early-age fatal conditions and work-related external causes do not influence the Gompertzian adherence for each population group that died due to adulthood onward biological causes.

While this study does not specifically address the timing of retirement benefit decisions in the presence of certain health conditions, it is crucial to understand that the possibility of early death can significantly influence various long-term decisions. Nielsen and Lomborg (2017) explored health-related decisions, noting that individuals with chronic conditions must monitor their emotions during decision-making to ensure they are not misled. This dynamic can lead to deviations from the normative framework, highlighting the need for more behavioral approaches to unravel the annuity puzzle.

In the next subsections, the *MWR* results for the six leading underlying causes of deaths of deaths will be presented, ordered from the lowest to the highest. The underlying cause of death corresponds to the health condition that starts the sequence of physiological events that ultimately lead the individual to death due to a complete depletion of the body's ability to sustain homeostasis (Clegg et al., 2013).

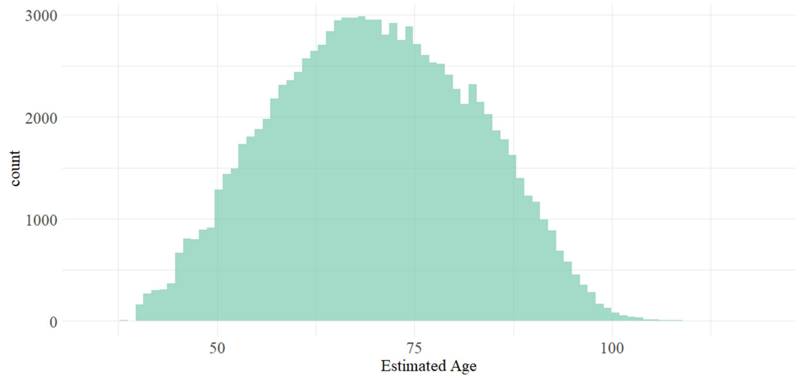
Figure 2 – Age at Death Distribution of the Main K-means Clusters in 2022 Brazilian Mortality Data.



4.2 Malignant Neoplasms

Figure 3 presents the distribution of ages at death from several types of malignant neoplasms, or cancers, which include a wide range of conditions affecting lymphoid and hematopoietic tissues and specific organs such as the breast, digestive organs, and respiratory system. The graph shows a peak around the age of seventy, indicating that the highest frequency of cancer-related deaths occurs early in the de-accumulating phase. Significant numbers start appearing in middle age, underscoring the widespread impact of cancer across these age groups.

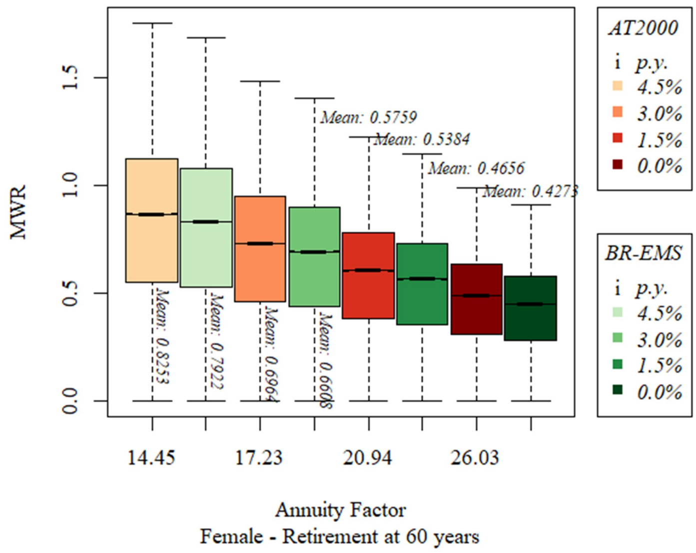
Figure 3 – Distribution of Deaths by Age – Female – Malignant Neoplasms



Consequently, the *MWR*, as presented in Figure 4, holds significant implications as it reflects the lowest mean life expectancy among the health conditions analyzed. In the actuarially fair scenario for the female version of the AT2000, a common life table in *EFPCs* schemes, the *MWR* shows a mean of 0.6964. This life table presents a life expectancy of 84.34 years at birth, a significant 14.4 years more than the 69.94 years simulated by the Gompertizian IBGE 2022 Female when aggravated in 127%. This data clearly illustrates a scenario where even overly fair annuities are unattractive.

This also results in an average Years of Life Lost (*YLL*) of 15.7 years compared to the corresponding birth life expectancy average derived from the life tables used for annuity calculations (BR-EMSsb-v.2021 and AT2000). The *YLL* is expected to be zero if the actual age of death coincides with the life expectancy estimated in the life table used for annuity calculations

Figure 4 – Distribution of Deaths by Age – Females – Malignant Neoplasms



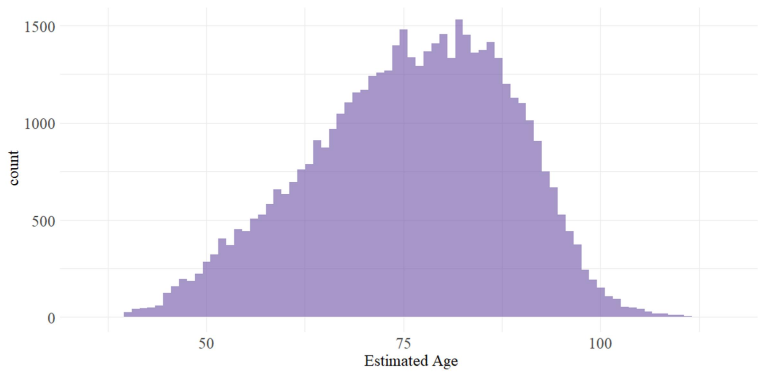
However, as several types of cancer may present symptoms only when it is too late, decisions based on this prospect may be ambiguous. The decision might vary on a case-by-case basis, relying upon one's previous knowledge of this condition before the retirement decision or, at the very least, on the known risk factors of which the individual is aware.

4.3 Diabetes mellitus and endocrine disorders

Figure 5 illustrates the age distribution at death from diabetes mellitus and endocrine disorders, which include coagulation defects, disorders of the thyroid, and obesity. It shows peaks around 75 to 85, suggesting that mortality from these conditions has a broad impact, affecting a wide age range.

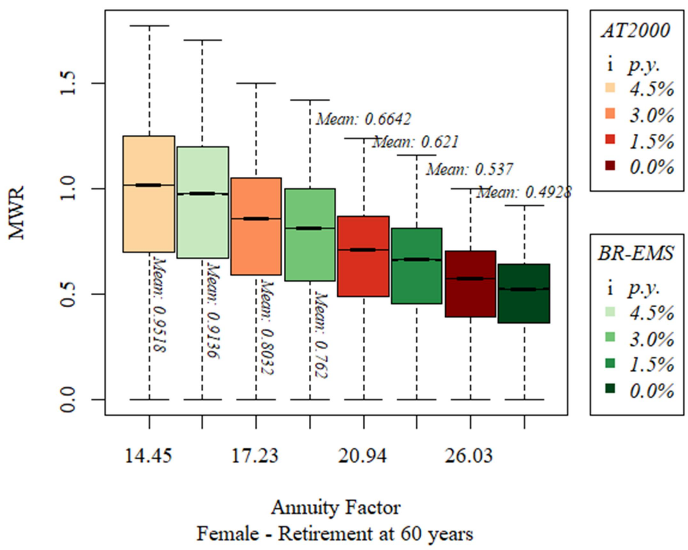
This broad impact reflects the chronic nature and long-term effects on the health of these diseases, which usually present an early onset, potentially steering the individual to non-lifelong benefits. Nonetheless, considering that diabetes mellitus is currently a well-treatable disease, premature mortality related to that might indicate a lack of healthcare access, making annuities potentially unaffordable.

Figure 5 – Distribution of Deaths by Age – Diabetes mellitus and endocrine disorders



The *MWR*, as presented in Figure 6, also has implications that reflect a shortened life expectancy prospect. In the actuarially fair scenario for the female version of BR-EMSsb-v.2021, the current standard for *EAPCs*, the *MWR* shows a mean of 0.762. This life table has a life expectancy at birth of 86.97 years, 11.1 years more than the 75.92 years estimated by the Gompertizian IBGE 2022 Female aggravated in 41%. The average *YLL* is 9.8 years.

Figure 6 – Distribution of Deaths by Age – Females – Diabetes mellitus and endocrine disorders

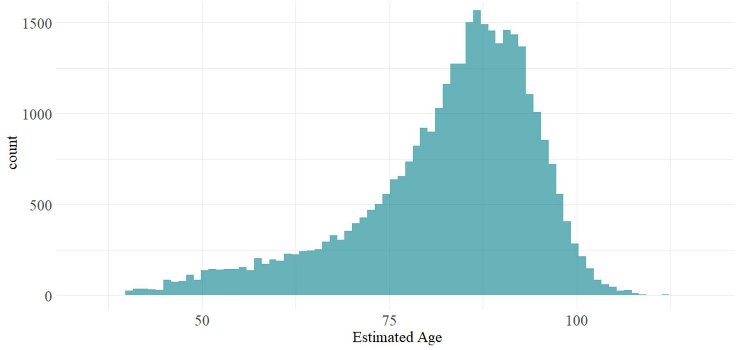


4.4 Cardiovascular Diseases

Figure 7 illustrates the distribution of ages at death from cardiovascular diseases. The distribution shows a pronounced peak at around 90 years, indicating a high mortality rate in older age ranges due to these conditions. The distribution starts to increase from age 50 and continues throughout the lifespan, highlighting the extensive impact of cardiovascular diseases.

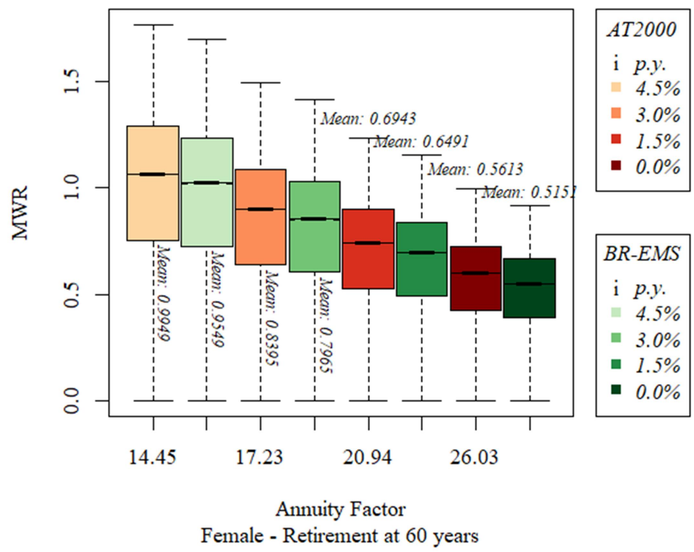
These diseases are major contributors to global morbidity and mortality, and they can significantly reduce life expectancy based on the severity of the disease, the effectiveness and timeliness of treatment, and personal lifestyle factors such as diet and exercise. Given the complexity and variety of health conditions related to the cardiovascular system, making decisions regarding retirement can be challenging. These decisions may not always be straightforward, as they could be influenced by numerous risk factors or congenital abnormalities that manifest later in life.

Figure 7 – Distribution of Deaths by Age – Cardiovascular Diseases



The *MWR*, as presented in Figure 8, indicates an unfavorable life expectancy prospect. In the actuarially fair scenario, the *MWR* is 0.8395 for the female version of AT2000 and 0.7965 for the female version of BR-EMSsb-v.2021. The Gompertizian IBGE 2022 Female estimate, with a 21% increase in the force of mortality, suggests a life expectancy of 77.75 years. The average *YLL* is 8.1 years.

Figure 8 – Distribution of Deaths by Age – Females – Cardiovascular Diseases

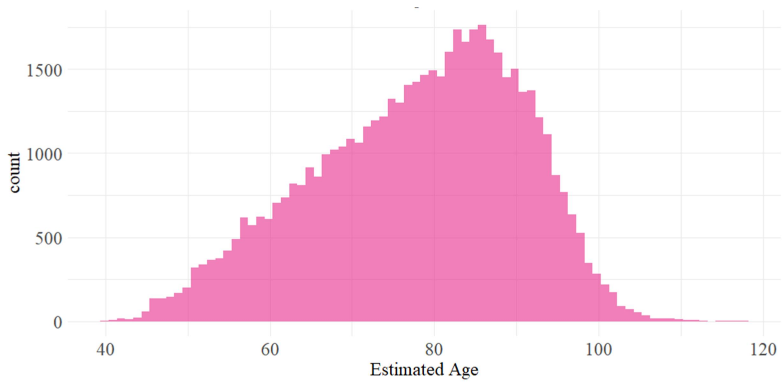


4.5 Infectious and Parasitic Diseases

Figure 9 illustrates the age distribution at death due to infectious and parasitic diseases, such as foodborne intoxications, hepatitis, human immunodeficiency virus, meningitis, syphilis, tuberculosis, and yellow fever. The histogram shows a peak around age 85, indicating a higher frequency of deaths in the elderly. The distribution ranges from age 40 to over one hundred, highlighting the variable impact across different life stages.

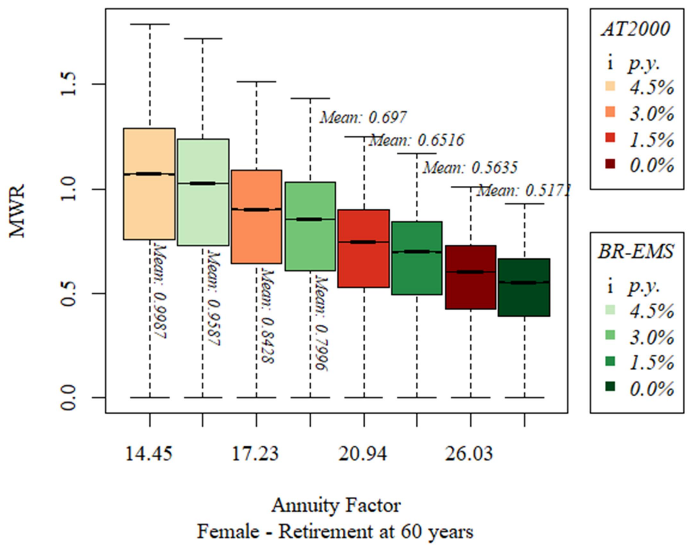
Often acquired in adulthood and known at retirement, these conditions can impact annuitization decisions. However, given the diverse nature of communicable diseases, their potential onset at older ages can introduce ambiguity, underscoring the importance of health factors in retirement planning.

Figure 9 – Distribution of Deaths by Age – Infectious and Parasitic Diseases



The *MWR*, as presented in Figure 10, also indicates a shorter life expectancy prospect, with equivalent results to cardiovascular diseases. In the actuarially fair scenario, the *MWR* is 0.8428 for the female version of AT2000 and 0.7996 for the female version of BR-EMSsb-v.2021. The Gompertizian IBGE 2022 Female estimate, with a 19% increase in the force of mortality, suggests a life expectancy of 77.95 years. The average *YLL* is 7.7 years.

Figure 10 – Distribution of Deaths by Age – Females – Infectious and Parasitic Diseases

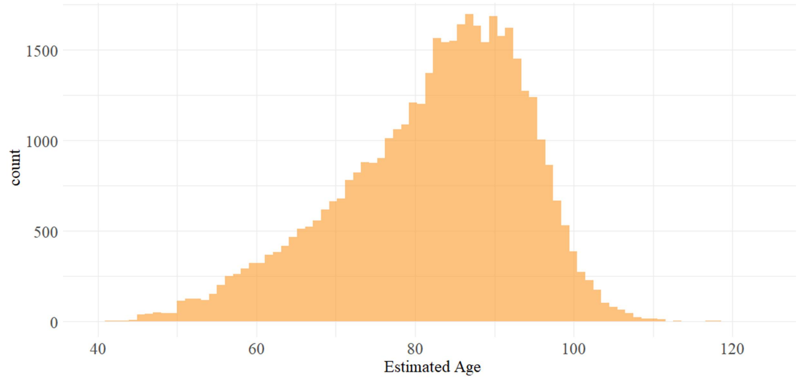


4.6 Respiratory Infections

Figure 22 illustrates the distribution of ages at death due to respiratory infections, covering acute lower and upper respiratory infections, emphysema, influenza, pneumonia, and COVID-19. The histogram shows a peak around age 90, indicating a higher frequency of deaths in the elderly due to frailties in the immunological system. Following communicable disease trends, the distribution range resembles the one seen in Infectious and Parasitic Diseases. However, the density before the age of sixty is significantly lower but slightly higher for centenarians.

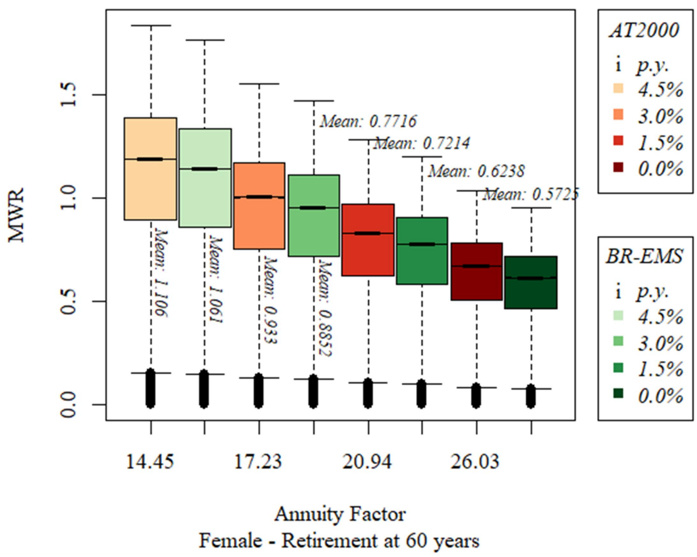
Also, unlike the other disease groups, respiratory infections are often acute, and death may occur quickly after their onset. Therefore, they are unlikely to steer the individual in a particular direction at retirement, as deaths due to respiratory infections might occur unpredictably.

Figure 11 – Distribution of Deaths by Age – Respiratory Infections



The *MWR* (Figure 12) reflects a close-to-average life expectancy prospect, with actuarially fair results pretty close to one. The *MWR* is 0.933 for the female version of AT2000 and 0.8852 for the female version of BR-EMSsb-v.2021. The Gompertizian IBGE 2022 Female estimate, with an 18% decrease in the force of mortality, indicates a life expectancy of 82.28 years. The average *YLL* is 3.4 years.

Figure 12 – Distribution of Deaths by Age – Females – Respiratory Infections

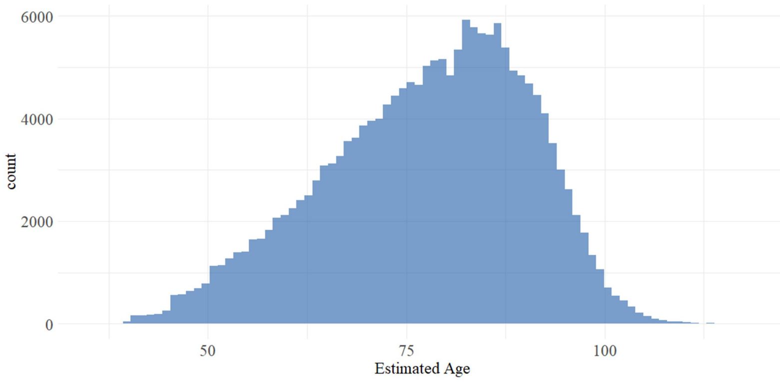


4.7 Neuropsychiatric conditions

Figure 13 presents a significant insight into the age distribution at death from neuropsychiatric conditions, such as Alzheimer's, Parkinson's, and sequelae from substance use disorders. The peaks around the age of ninety underscore these conditions' profound and long-term impact on health and quality of life. Importantly, these distributions also reveal that these diseases affect a broad age range, further underlining their significance.

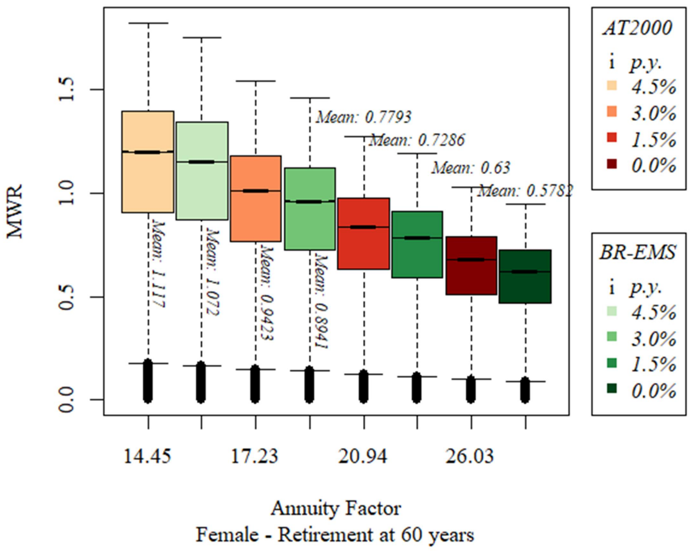
This broad impact reflects the chronic nature of these diseases and their long-term effects on health, often necessitating more financial resources to cover healthcare expenses. It may add complexity to the retirement decision-making process, particularly in cases of early disease onset or when significant risk factors are known. This intricate prospect may lead to the need to prepare arrangements against potential future daily life impacts, which often impact the routine of the retiree's family.

Figure 13 – Distribution of Deaths by Age – Neuropsychiatric conditions



The *MWR*, as presented in Figure 14, also reflects a close-to-average life expectancy prospect, with actuarially fair results also relatively close to one. The *MWR* is also 0.9423 for the female version of AT2000 and 0.8941 for the female version of BR-EMSsb-v.2021. The Gompertizian IBGE 2022 Female estimate, with a 21% decrease in the force of mortality, indicates a life expectancy of 82.28 years. The average *YLL* is 2.9 years.

Figure 14 – Distribution of Deaths by Age – Females – Neuropsychiatric conditions



5 – Final Comments

This study integrates discussions on health conditions and their extensive implications for retirement planning, particularly within Brazil's Annuity Puzzle retirement context. The findings highlight the significant impact of health conditions on retirement decisions, as although an annuity does not need to be fair, great deviation from this basis may lead the individual to opt for non-lifelong benefits. Also, some health conditions may require more financial liquidity or a greater disposition to leave a bequest.

The results also identify discrepancies between life expectancy in annuity pricing and actual mortality data, which may lead to adverse selection. Exploring the *MWR* and its implications in the context of health conditions reveals the complexities of retirement planning. A deeper understanding of this interaction might aid in the decision-making process.

Strategies tailored to health risks and their corresponding prospective life expectancy can lead to a more appealing lifelong benefit, mitigating the adverse selection that derives from the assumption that all population groups follow the same force of mortality. Detailed analyses of mortality distributions across various health conditions, approached in this study for the main underlying causes of death in Brazil in 2022, reveal their significant impact across various age groups, particularly the elderly. This underscores the need for a non-one-size-fits-all retirement setting, integrated with health assessment.

Moreover, this study underscores the importance of understanding and addressing disparities in annuity outcomes among different populations. Although Brazilian socioeconomic disparities were not the focus of this study, the differences in life expectancy data between Brazilian populations and those used in actuarial tables highlight how inequality also plays a role in this process, leaving not only a majority of the Brazilian population outside the realm of the *RPC* but also potential annuity buyers astray due other pressing matters.

The insights from this study advocate for a multidimensional approach to health conditions and retirement planning that integrates medical and social strategies. This comprehensive approach aims to improve financial security throughout retirement. By aligning health and financial strategies with the population's health profiles and socioeconomic conditions, the Brazilian *RPC* can establish a more equitable framework for retirement planning. Such a holistic strategy is vital for tackling the Brazilian population's complex health challenges and fostering a healthier and more financially secure society.

6 – References

Ai, J., Brockett, P. L., Golden, L. L., & Zhu, W. (2017). Health State Transitions and Longevity Effects on Retirees' Optimal Annuitization. *Journal of Risk and Insurance*, *84*(S1), 319–343. https://doi.org/10.1111/jori.12168.

Alexandrova, M., & Gatzert, N. (2019). What Do We Know About Annuitization Decisions? *Risk Management and Insurance Review*, *22*(1), 57–100. https://doi.org/10.1111/rmir.12115.

Ameriks, J., Caplin, A., Laufer, S., & Van Nieuwerburgh, S. (2011). The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives. *Journal of Finance*, *66*(2), 519–561. https://doi.org/10.1111/j.1540-6261.2010.01641.x.

Arvanitis, A., Papadatou-Pastou, M., & Hantzi, A. (2019). Agreement in the ultimatum game: An analysis of interpersonal and intergroup context on the basis of the consensualistic approach to negotiation. *New Ideas in Psychology*, *54*(January 2019), 15–26. https://doi.org/10.1016/j.newideapsych.2018.12.005.

Asher, A. (2021). Supply Side Resistance to Lifetime Annuities. *SSRN Electronic Journal*, *November*, 1–18. https://doi.org/10.2139/ssrn.3981196.

Assaf Neto, A. (2006). *Matemática financeira e suas aplicações* (9th ed.). Atlas.

Barr, N., & Diamond, P. (2006). The economics of pensions. *Oxford Review of Economic Policy*, *22*(1), 15–39. https://doi.org/10.1093/oxrep/grj002.

Benartzi, S., Previtero, A., & Thaler, R. H. (2011). Annuitization puzzles. *Journal of Economic Perspectives*, *25*(4), 143–164. https://doi.org/10.1257/jep.25.4.143.

Berger, R., Rauhut, H., Prade, S., & Helbing, D. (2012). Bargaining over waiting time in ultimatum game experiments. *Social Science Research*, *41*(2), 372–379. https://doi.org/10.1016/j.ssresearch.2011.09.009.

Bowers, N. L. J., Gerber, H. U., Hickman, J. C., Jones, D. A., & Nesbitt, C. J. (1997). Actuarial

Mathematics. In *The American Mathematical Monthly* (2nd ed., Vol. 93, Issue 1). The Society of Actuaries. https://doi.org/10.1080/00029890.1986.11971867.

Boyle, P., Tan, K. S., Wei, P., & Zhuang, S. C. (2022). Annuity and insurance choice under habit formation. *Insurance: Mathematics and Economics*, *105*, 211–237. https://doi.org/10.1016/j.insmatheco.2022.04.003.

Brown, J. R. (2001). Private pensions, mortality risk, and the decision to annuitize. *Journal of Public Economics*, *82*(1), 29–62. https://doi.org/10.1016/S0047-2727(00)00152-3.

Brown, J. R., Kapteyn, A., Luttmer, E. F. P., & Mitchell, O. S. (2013). Decision Complexity as a Barrier to Annuitization. In *SSRN Electronic Journal* (DP 06/2013-058; Netspar Discussion Papers). https://doi.org/10.2139/ssrn.2357370.

Bütler, M., & Teppa, F. (2007). The choice between an annuity and a lump sum: Results from Swiss pension funds. *Journal of Public Economics*, *91*(10), 1944–1966. https://doi.org/10.1016/j.jpubeco.2007.09.003.

Campani, C. H., Costa, T. R. D., Martins, F. G. L., & Azambuja, S. de. (2020). Planos PGBL e VGBL e Previdência Privada: Uma Análise do Mercado Brasileiro. *Sociedade, Contabilidade e Gestão*, *15*(1), 122–141. https://doi.org/10.21446/scg_ufrj.v0i0.18360.

Cappelletti, G., Guazzarotti, G., & Tommasino, P. (2013). What determines annuity demand at retirement? *The Geneva Papers on Risk and Insurance: Issues and Practice*, *38*(4), 777–802. https://doi.org/10.1057/gpp.2012.43.

Chai, J., Horneff, W., Maurer, R., & Mitchell, O. S. (2011). Optimal portfolio choice over the life cycle with flexible work, endogenous retirement, and lifetime payouts. *Review of Finance*, *15*(4), 875–907. https://doi.org/10.1093/rof/rfr016.

Chalmers, J., & Reuter, J. (2012). How do retirees value life annuities? Evidence from public employees. *Review of Financial Studies*, *25*(8), 2601–2634. https://doi.org/10.1093/rfs/hhs057.

Chen, A., & Rach, M. (2022). The tontine puzzle. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4106903.

Clegg, A., Young, J., Iliffe, S., Rikkert, M. O., & Rockwood, K. (2013). Frailty in elderly people. *The Lancet*, *381*(9868), 752–762. https://doi.org/10.1016/S0140-6736(12)62167-9.

d'Albis, H., Attanasi, G., & Thibault, E. (2020). An experimental test of the under-annuitization puzzle with smooth ambiguity and charitable giving. *Journal of Economic Behavior and Organization*, *180*, 694–717. https://doi.org/10.1016/j.jebo.2019.09.019.

Davidoff, T., Brown, J. R., & Diamond, P. A. (2005). Annuities and Individual Welfare. *The American Economic Review*, *95*(5), 1573–1590. https://doi.org/10.1257/000282805775014281.

De Villiers-Strijdom, J., & Krige, N. (2023). Living annuity satisfaction. *Journal of Economic and Financial Sciences*, *16*(1), 1–12. https://doi.org/10.4102/jef.v16i1.841.

Dickson, D. C. M., Hardy, M. R., & Waters, H. R. (2009). Actuarial Mathematics for Life Contingent Risks. In *International Series on Actuarial Science* (1st ed.). Cambridge University Press.

Dus, I., Maurer, R., & Mitchell, O. S. (2005). Betting on Death and Capital Markets in Retirement : a Shortfall Risk Analysis of Life Annuities. *National Bureau of Economic Research No. W11271*, 1–39.

Goedde-Menke, M., Lehmensiek-Starke, M., & Nolte, S. (2014). An empirical test of competing hypotheses for the annuity puzzle. *Journal of Economic Psychology*, *43*, 75–91. https://doi.org/10.1016/j.joep.2014.04.001.

Gompertz, B. (1825). On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. *Philosophical Transactions of the Royal Society of London*, *115*(115), 513–583. http://www.jstor.com/stable/107756%0A.

Guillemette, M. A., Martin, T. K., Cummings, B. F., & James, R. N. (2015). Determinants of the stated probability of purchase for longevity insurance. In *Geneva Papers on Risk and Insurance: Issues and Practice*. Palgrave Macmillan Ltd. https://doi.org/10.1057/gpp.2015.26.

Gupta, A., & Li, L. (2007). Integrating long-term care insurance purchase decisions with saving and investment for retirement. *Insurance: Mathematics and Economics*, *41*(3), 362–381. https://doi.org/10.1016/j.insmatheco.2006.11.008.

Güth, W., & Van Damme, E. (1998). Information, Strategic Behavior, and Fairness in Ultimatum Bargaining: An Experimental Study. *Journal of Mathematical Psychology*, *42*(2–3), 227–247. https://doi.org/10.1006/jmps.1998.1212.

Hagen, J. (2015). The determinants of annuitization: evidence from Sweden. *International Tax and Public Finance*, *22*(4). https://doi.org/10.1007/s10797-015-9360-5.

Han, N. W., & Hung, M. W. (2021). The annuity puzzle and consumption hump under ambiguous life expectancy. *Insurance: Mathematics and Economics*, *100*, 76–88. https://doi.org/10.1016/j.insmatheco.2021.04.011.

Holzmann, R., Paul, R., & Dorfman, M. (2008). Pension Systems and Reform Conceptual Framework. *World Bank SP Discussion Paper*, *0824*, 31. https://econpapers.repec.org/paper/wbkhdnspu/46175.htm.

Horneff, V., Maurer, R., Mitchell, O. S., & Rogalla, R. (2015). Optimal life cycle portfolio choice with variable annuities offering liquidity and investment downside protection. *Insurance: Mathematics and Economics*, *63*, 91–107. https://doi.org/10.1016/j.insmatheco.2015.03.031.

Horneff, W. J., Maurer, R. H., Mitchell, O. S., & Dus, I. (2008b). Following the rules: Integrating asset allocation and annuitization in retirement portfolios. *Insurance: Mathematics and Economics*, *42*(1), 396–408. https://doi.org/10.1016/j.insmatheco.2007.04.004.

Horneff, W. J., Maurer, R. H., & Stamos, M. Z. (2008a). Life-cycle asset allocation with annuity markets. *Journal of Economic Dynamics and Control*, *32*(11), 3590–3612. https://doi.org/10.1016/j.jedc.2008.01.007.

Hu, W. Y., & Scott, J. S. (2007). Behavioral obstacles in the annuity market. *Financial Analysts Journal*, *63*(6), 71–82. https://doi.org/10.2469/faj.v63.n6.4928.

Humes, A. F. P. de C., Melo, I. S. H. de;, Yoshida, L. K., & Martins, W. T. (1984). Noções de Cálculo Numérico. In J. M. Braga (Ed.), *Fundamentos de Matemática*. McGraw-Hill.

Hwang, Y., Chang, S., & Yang, S. (2012). Measuring the consequences of pension reform applying liquidation and longevity considerations. *Journal of Economic Policy Reform*, *15*(3), 243–255. https://doi.org/10.1080/17487870.2012.696417.

Jeong, S. Y., Owadally, I., Haberman, S., & Wright, D. (2023). Subjective survival beliefs and the life-cycle model. *SSRN Electronic Journal*, 1–35. https://doi.org/10.2139/ssrn.4637240.

Jong, J. De, & Robinson, J. (2022). Is the Annuity Puzzle Really So Puzzling? An Analysis of the Consumer Lifetime Annuitization Decision in a Low Interest Rate World. *SSRN Electronic Journal*, 1–48.

Kling, A., Richter, A., & Ruß, J. (2014). Annuitization behavior: Tax incentives vs. product design. *ASTIN Bulletin: The Journal of the IAA*, *44*(3), 535–558. https://doi.org/10.1017/asb.2014.17.

Koijen, R. S. J. J., Van Nieuwerburgh, S., Yogo, M., Nieuwerburgh, S. V. A. N., & Yogo, M. (2016). Health and Mortality Delta: Assessing the Welfare Cost of Household Insurance Choice. *The Journal of Finance*, *71*(2), 957–1009. https://doi.org/10.1111/jofi.l2273.

Korankye, T., Pearson, B., & Salehi, H. (2023). The nexus between investor sophistication and annuity insurance ownership: evidence from FINRA's National Financial Capability Study. *Managerial Finance*, *49*(2), 398–419. https://doi.org/10.1108/MF-04-2022-0169.

Lambregts, T. R., & Schut, F. T. (2020). Displaced, disliked and misunderstood: A systematic review of the reasons for low uptake of long-term care insurance and life annuities. *Journal of the Economics* *of Ageing*, *17*(January), 100236. https://doi.org/10.1016/j.jeoa.2020.100236.

Liang, X., Peng, X., & Guo, J. (2014). Optimal investment, consumption and timing of annuity purchase under a preference change. *Journal of Mathematical Analysis and Applications*, *413*(2), 905–938. https://doi.org/10.1016/j.jmaa.2013.12.036.

Lockwood, L. M. (2012). Bequest motives and the annuity puzzle. *Review of Economic Dynamics*, *15*(2), 226–243. https://doi.org/10.1016/j.red.2011.03.001.

Look, S., Guo, T., & Cheng, Y. (2023). Solving the Puzzle within the Annuity Puzzle: Incorporating Irrevocability Aversion into Annuity Choice. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4555546.

MacMinn, R. D., & Ren, Y. (2023). The Life Annuity Puzzles? *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4324621.

Milevsky, M. A. (1998). Optimal Asset Allocation towards the End of the Life Cycle: To Annuitize or Not to Annuitize? *The Journal of Risk and Insurance*, *65*(3), 401–426. https://doi.org/10.2307/253657.

Milevsky, M. A., & Young, V. R. (2007). Annuitization and asset allocation. *Journal of Economic Dynamics and Control*, *31*(9), 3138–3177. https://doi.org/10.1016/j.jedc.2006.11.003

Mindlin, D. (2021). The 'Annuity Puzzle' and Model Portfolios. *SSRN Electronic Journal*, 1–8. https://doi.org/10.2139/ssrn.3740892.

Ministério da Previdência Social. (2023a). Acompanhamento Estatístico da Evolução dos Segmentos Aberto e Fechado de Previdência Complementar. In *Relatório Gerencial de Previdência Complementar: Vol. 3º Trimest*. https://www.gov.br/previdencia/pt-br/assuntos/previdencia-complementar/dados-abertos-de-previdencia-complementar.

Ministério da Previdência Social. (2023b). Valor de aposentadorias urbanas emitidas, por grupo de espécies, segundo as Grandes Regiões e Unidades da Federação - Acumulado no ano - 2020/2022. In *Versão Online AEPS 2022*. https://www.gov.br/previdencia/pt-br/assuntos/previdencia-social/arquivos/copy_of_onlinte-aeps-2022-/secao-i-beneficios/subsecao-b-beneficios-emitidos/capitulo-8-2013-aposentadorias/8-3-valor-de-aposentadorias-urbanas-emitidas-por-grupos-de-especies-segun.

Modigliani, F. (1986). Life Cycle, Individual Thrift, and the Wealth of Nations. *The American Economic Review*, *76*(3), 297–313. https://doi.org/10.1126/science.234.4777.704

Nielsen, B. K., & Lomborg, K. (2017). The Role of Illness Perceptions on Health-Related Decision-Making—A Focus Group Study of Patients with Long-Term Conditions. *Psychology*, *08*(09), 1267–1279. https://doi.org/10.4236/psych.2017.89082.

O'Dea, C., & Sturrock, D. (2023). Survival Pessimism And The Demand For Annuities. *The Review of Economics and Statistics*, *105*(2), 1–16. https://doi.org/doi.org/10.1162/rest_a_01048.

Pang, G., & Warshawsky, M. (2010). Optimizing the equity-bond-annuity portfolio in retirement: The impact of uncertain health expenses. *Insurance: Mathematics and Economics*, *46*(1), 198–209. https://doi.org/10.1016/j.insmatheco.2009.08.009.

Pashchenko, S. (2013). Accounting for non-annuitization. *Journal of Public Economics*, *98*, 53–67. https://doi.org/10.1016/j.jpubeco.2012.11.005.

Pashchenko, S., & Porapakkarm, P. (2022). Value of life and annuity demand. *Journal of Risk and Insurance*, *89*(2), 371–396. https://doi.org/10.1111/jori.12370.

Pashchenko, S., & Porapakkarm, P. (2024). Accounting for Social Security Claiming Behavior. *International Economic Review*, *65*(1), 505–545. https://doi.org/10.1111/iere.12658.

Peijnenburg, K., Nijman, T., & Werker, B. J. M. (2016). The annuity puzzle remains a puzzle. *Journal of Economic Dynamics and Control*, *70*, 18–35. https://doi.org/10.1016/j.jedc.2016.05.023.

Poterba, J., Venti, S., & Wise, D. (2011). The Composition and Drawdown of Wealth in Retirement. *The*

Journal of Economic Perspective, *25*(4), 95–118. https://doi.org/10.1257/jep.27.4.219.

Previtero, A. (2014). Stock market returns and annuitization. *Journal of Financial Economics*, *113*(2), 202–214. https://doi.org/10.1016/j.jfineco.2014.04.006.

Reichling, F., & Smetters, K. (2015). Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs. *The American Economic Review*, *105*(11), 3273–3320. https://doi.org/10.1257/aer.20131584.

Rusconi, R. (2008). National Annuity Markets FEATURES AND IMPLICATIONS. In *OECD Working Papers on Insurance and Private Pensions* (Issue 24). OECD publishing. https://doi.org/10.1787/240211858078.

Salisbury, L. C., & Nenkov, G. Y. (2016). Solving the annuity puzzle: The role of mortality salience in retirement savings decumulation decisions. *Journal of Consumer Psychology*, *26*(3), 417–425. https://doi.org/10.1016/j.jcps.2015.10.001.

Schreiber, P., & Weber, M. (2016). Time inconsistent preferences and the annuitization decision. *Journal of Economic Behavior and Organization*, *129*, 37–55. https://doi.org/10.1016/j.jebo.2016.06.008

Shu, S. B., Zeithammer, R., & Payne, J. W. (2016). Consumer preferences for annuity attributes: Beyond net present value. *Journal of Marketing Research*, *53*(2), 240–262. https://doi.org/10.1509/jmr.13.0004.

Sutcliffe, C. (2015). Trading death: The implications of annuity replication for the annuity puzzle, arbitrage, speculation and portfolios. *International Review of Financial Analysis*, *38*, 163–174. https://doi.org/10.1016/j.irfa.2014.10.010.

Tolley, H. D., Barnes, J. M., & Freeman, M. D. (2016). Survival Analysis. In *Forensic Epidemiology: Principles and Practice* (pp. 261–284). Elsevier Inc. https://doi.org/10.1016/B978-0-12-404584-2.00010-0

Valdez, E. A., Piggott, J., & Wang, L. (2006). Demand and adverse selection in a pooled annuity fund. *Insurance: Mathematics and Economics*, *39*(2), 251–266. https://doi.org/10.1016/j.insmatheco.2006.02.011.

Wang, T., & Young, V. R. (2012a). Maximizing the utility of consumption with commutable life annuities. *Insurance: Mathematics and Economics*, *51*(2), 352–369. https://doi.org/10.1016/j.insmatheco.2012.06.002.

Wang, T., & Young, V. R. (2012b). Optimal commutable annuities to minimize the probability of lifetime ruin. *Insurance: Mathematics and Economics*, *50*(1), 200–216. https://doi.org/10.1016/j.insmatheco.2011.10.012.

Webb, D. C. (2009). Asymmetric information, long-term care insurance, and annuities: The case for bundled contracts. *Journal of Risk and Insurance*, *76*(1), 53–85. https://doi.org/10.1111/j.1539-6975.2009.01288.x.

World Bank Group. (2024). Gini Index Data. In *Data Bank Microdata*. https://data.worldbank.org/indicator/SI.POV.GINI?most_recent_year_desc=true&skipRedirection=true&view=map.

Wu, S., Stevens, R., & Thorp, S. (2015). Cohort and target age effects on subjective survival probabilities: Implications for models of the retirement phase. *Journal of Economic Dynamics and Control*, *55*, 39–56. https://doi.org/10.1016/j.jedc.2015.03.003.

Yaari, M. E. (1965). Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. *The Review of Economic Studies*, *32*(2), 137–150. https://doi.org/10.2307/2296058.

Zhang, J., Purcal, S., & Wei, J. (2021). Optimal life insurance and annuity demand under hyperbolic discounting when bequests are luxury goods. *Insurance: Mathematics and Economics*, *101*, 80–90. https://doi.org/10.1016/j.insmatheco.2020.07.003.